

1. This problem is about the Ideal Fermi Gas. Show that the free energy of an ideal Fermi gas of  $N$  particles can be written as

$$F = N\mu - kT \sum_r \log [1 + e^{\beta(\mu - \epsilon_r)}], \quad (1)$$

where  $\epsilon_r$  is the energy of the  $r$ 'th level, and where

$$kT \frac{\partial}{\partial \mu} \left[ \sum_r \log [1 + e^{\beta(\mu - \epsilon_r)}] \right] = N \quad (2)$$

determines  $\mu$ . Consider a metal which contains a set of states that could be completely filled by  $2N$  electrons (there are  $N$  states, two spin degeneracies) but which contains only  $2N - N'$  electrons. Show that the contributions of these electrons, say to the energy  $E$  as a function of temperature, is equivalent to that of a Fermi gas of 'holes' for which the energy levels are  $-\epsilon_r$  and the chemical potential is  $-\mu$ .

2. This problem is about a Chain with a Basis. Consider a one-dimensional chain with alternating masses and springs. All of the springs have constant  $K$ , but there are two alternating masses, of mass  $M$  and  $m$ .
- (a) Write the equation of motion for the masses. Assume that the masses behave as  $x_n = Ae^{iqn - i\omega t}$  for  $n$  odd and  $x_n = Be^{iqn - i\omega t}$  for  $n$  even. Find the normal modes for a chain of  $N$  big masses and  $N$  little masses. The modes divide into two branches. Sketch them. In crystals of this type, the upper group of modes is called the optical mode and the lower group is called the acoustic mode.
- (b) Find the specific heat of the chain. You must include both optical and acoustic branches.