944 QUANTUM MECHANICS - Homework #5. Due: Tuesday, December 2, 2008.

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1. This problem is about neutron-proton scattering. Consider the simple model of S-wave scattering with a square well of depth V_0 and width R discussed in class.

   (a) Calculate the phase shift $\delta_0(k)$.

   (b) Calculate the scattering length and effective range.

   (c) Treating the two independent S-wave channels, $^1S_0$ and $^3S_1$, separately, fit V_0 and R to the scattering length and effective range.

   (d) (Extra Credit) Go to the web site NN-OnLine and obtain the "experimental" neutron-proton S-wave phase shifts as a function of lab energy T (in tabular format). Use PWA: PWA93. Convert to center-of-mass momentum k using

$$k = \frac{M_N}{2}\sqrt{T} \tag{1}$$

   where M_N is the nucleon mass. Then plot the S-wave phase shifts in the square well model and compare to experiment.

2. This problem is about the optical theorem (Merzbacher, chapter 20, excercises 20.8,20.9). The unitarity of the S matrix can be expressed as

$$\sum_n S^*_{nl} S_{nj} \;=\; \delta_{lj} \; . \tag{2}$$

   (a) Show that this implies the following constraint on the T matrix:

$$2\pi \sum_n \delta(E_n - E_r) T^*_{nr} T_{nj} \;=\; i(T_{rj} - T^*_{jr}) \; . \tag{3}$$

   This is an expression of the optical theorem.

   (b) Now derive the optical theorem directly from

$$\langle k|\tilde{T}(t,-\infty)|s\rangle \;=\; \delta_{ks} \;+\; \frac{T_{ks} e^{i\omega_{ks}t+\alpha t}}{\hbar(-\omega_{ks}+i\alpha)} \; , \tag{4}$$

   using conservation of probability.

   (c) Show that the first Born approximation violates the optical theorem. Explain this failure and show how it can be remedied by including the second Born approximation for the forward scattering amplitude.

3. This problem is about separable potentials (Merzbacher, chapter 20, problem 5). If the nonlocal separable potential

$$\langle \mathbf{r}'|V|\mathbf{r}''\rangle \;=\; \lambda u(r')u(r'') \tag{5}$$

   is given, work out explicitly and solve the integral equation for $|\Psi^+\rangle$. Obtain the scattering amplitude, and discuss the Born series for this exponential.