944 QUANTUM MECHANICS - Homework #3. Due: Friday, October 17, 2008.

---

1. This problem is about oscillator strengths in the hydrogen atom (Merzbacher chapter 19, exercise 19.5). For a hydrogen atom that is initially in the 1S ground state, compute the oscillator strengths to the lowest three excited states. What fraction of the total do these oscillator strengths add up to? (Use the z coordinate, rather than x, to simplify the calculations.)

2. This problem is about a generalization of the Thomas-Reiche-Kuhn (TRK) sum rule (Merzbacher chapter 19, problem 3). By considering the double commutator

$$[[H, e^{ik_j r_j}], e^{-ik_j r_j}] \tag{1}$$

obtain as a generalization of the TRK sum rule the formula

$$\sum_n (E_n - E_s) |\langle n| e^{ik_j r_j} |s\rangle|^2 \;=\; \frac{\hbar^2 k^2}{2m} \; . \tag{2}$$

Specify the conditions on the Hamiltonian, H, required for the validity of the sum rule.

3. This problem is about perturbing the harmonic oscillator (Sakurai chapter 5, problem 24). Consider a particle bound in a simple harmonic oscillator potential. Initially ($t < 0$), it is in the ground state. At $t = 0$, a perturbation of the form

$$H'(x,t) \;=\; Ax^2 e^{-t/\tau} \tag{3}$$

is switched on. Using time-dependent perturbation theory, calculate the probability that, after a sufficiently long time ($t \gg \tau$), the system will have made a transition to a given excited state. Consider all final states.