944 QUANTUM MECHANICS - Homework #2. Due: Friday, October 3, 2008.

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1. This problem is about the Variational Approximation for the potential

$$V(x) \;=\; g|x|, \tag{1}$$

in one spatial dimension. This system is a one-dimensional analogue of a potential that confines quarks and antiquarks to give the charmonium, bottomonium, *etc.* spectrum. (See Merzbacher Ch7. and Ch.8.) It will be helpful if you review the exact solution for this potential.

   (a) Calculate the variational estimate for the ground-state energy of this potential (with $g > 0$), using the *triangular* trial function:

$$\psi(x) \;=\; C\,(\alpha - |x|) \quad \text{for} \quad |x| \le \alpha \quad \text{and} \quad \psi(x) \;=\; 0 \quad \text{for} \quad |x| > \alpha\;. \tag{2}$$

   Which form of "I" obtained in class is it safest to use here? How good is the estimate? Compare your result with the Gaussian trial function worked out by Merzbacher.

   (b) By using odd trial functions with only one node, at the origin, obtain numerical estimates for the first excited state and compare with the exact value of E_1. Can you proceed to the second excited state and calculate a variational energy estimate for this state?

2. This problem is about Rabi's formula (see Sakurai, chapter 5 and problem 5.30). Consider a two-level system with $E_1 < E_2$. There is a time-dependent potential that connects the two levels as follows:

$$V_{11} \;=\; V_{22} \;=\; 0, \qquad V_{12} \;=\; \gamma\, e^{i\omega t}, \qquad V_{21} \;=\; \gamma\, e^{-i\omega t} \quad (\gamma \text{ real})\;, \tag{3}$$

where

$$V_{nm} \;\equiv\; \langle n|\, V\, |m\rangle\;, \tag{4}$$

and

$$H_0\,|n\rangle \;=\; E_n\,|n\rangle\;. \tag{5}$$

At $t = 0$, it is known that only the lower level is populated; that is, $c_1(0) = 1$ and $c_2(0) = 0$, where

$$\left|\tilde{\psi}(t)\right\rangle \;=\; \sum_n\, |n\rangle\,\langle n|\left|\tilde{\psi}(t)\right\rangle \;\equiv \sum_n\, c_n(t)\,|n\rangle\;. \tag{6}$$

   (a) Find $|c_1(t)|^2$ and $|c_2(t)|^2$ for $t > 0$ by solving for the quantum dynamics *exactly*.

   (b) Solve the same problem but now using time-dependent perturbation theory to lowest non-vanishing order. Compare the two approaches for small values of γ. Treat the following two cases separately: (i) ω very different from ω_{12} and (ii) ω very close to ω_{12}.