944 QUANTUM MECHANICS - Homework #1. Due: Friday, September 19, 2008.

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1. This problem is about time-independent perturbation theory.

   (a) Continuing where we left off in class, construct the eigenenergies, E_n, and eigenstates, $|n\rangle$, to $O(\lambda^3)$ and $O(\lambda^4)$ in perturbation theory.

   (b) Compare the eigenenergies found above at $O(\lambda^3)$ and $O(\lambda^4)$ in perturbation theory to the exact eigenenergies for the case of the ammonia molecule in the presence of an electric dipole moment (discussed in class), and discuss your results. In perturbation theory consider only the case of a weak edm.

2. This problem is about the variational method.

   Consider a particle in the potential

$$\hat{V}(\hat{u}) = \frac{m\omega^2}{2\alpha^2}\left(e^{-\alpha\hat{u}} - 1 \right)^2 . \tag{1}$$

   This is a special case of the Morse potential (P.M.Morse *Phys.Rev.***34**,57 (1929)) which gives a useful approximation of the interactions of a diatomic molecule. The variable $\hat{u}$ is the distance between atoms minus the equilibrium separation (i.e. $\hat{u} = r - r_e$).

   (a) Show that with $\hbar = m = \alpha = 1$ and $\omega = 10$, the true ground state energy of a particle in the Morse potential is $E_0 = 39/8$. Plot the form of the potential.

   (b) Find a variational wave function that comes within 5% of the true ground state energy.