**944 QUANTUM MECHANICS** - Homework #7. Due: Friday, December 14, 2007.

---

1. This problem is about the optical theorem (Merzbacher, chapter 20, excercises 20.8,20.9). The unitarity of the S matrix can be expressed as

$$\sum_n S^*_{nl} S_{nj} \;=\; \delta_{lj} \; . \tag{1}$$

   (a) Show that this implies the following constraint on the T matrix:

$$2\pi \sum_n \delta(E_n - E_r) T^*_{nr} T_{nj} \;=\; i(T_{rj} - T^*_{jr}) \; . \tag{2}$$

   This is an expression of the optical theorem.

   (b) Now derive the optical theorem directly from

$$\langle k|\tilde{T}(t, -\infty)|s\rangle \;=\; \delta_{ks} \;+\; \frac{T_{ks} e^{i\omega_{ks} t + \alpha t}}{\hbar(-\omega_{ks} + i\alpha)} \; , \tag{3}$$

   using conservation of probability.

   (c) Show that the first Born approximation violates the optical theorem. Explain this failure and show how it can be remedied by including the second Born approximation for the forward scattering amplitude.

2. This problem is about separable potentials (Merzbacher, chapter 20, problem 5). If the nonlocal separable potential

$$\langle \mathbf{r}'|V|\mathbf{r}''\rangle \;=\; \lambda u(r')u(r'') \tag{4}$$

   is given, work out explicitly and solve the integral equation for $|\Psi^+\rangle$. Obtain the scattering amplitude, and discuss the Born series for this exponential.

3. This problem is about two particle wavefunctions which do not exhibit correlations (Merzbacher, chapter 15, exercise 15.19). Show that the two-particle state with sharp momenta $\mathbf{p_1}$ and $\mathbf{p_2}$, corresponding to the plane wave function

$$\psi(\mathbf{r_1}, \mathbf{r_2}) \;=\; \exp\left(\frac{i}{\hbar} p^1_i r^1_i\right) \exp\left(\frac{i}{\hbar} p^2_i r^2_i\right) \tag{5}$$

   is also separable when it is canonically transformed into $\psi(\mathbf{r}, \mathbf{R})$.

4. This problem is about entangled states (Merzbacher, chapter 15, exercise 15.21). Check that the amplitude

$$\psi(x_1, x_2) \;=\; (2\pi\hbar)^{-1}\, \delta(x_1 - x_2 - a) \tag{6}$$

   where $a$ is a positive constant, is entangled by making a Fourier expansion in terms of momentum eigenfunctions or any other complete set of one-particle basis functions.