1. This problem is about Rabi’s formula (see Sakurai, chapter 5 and problem 5.30). Consider a two-level system with $E_1 < E_2$. There is a time-dependent potential that connects the two levels as follows:

$$V_{11} = V_{22} = 0, \quad V_{12} = \gamma e^{i\omega t}, \quad V_{21} = \gamma e^{-i\omega t} \quad (\gamma \text{ real}),$$

(1)

where

$$V_{nm} \equiv \langle n | V | m \rangle,$$

(2)

and

$$H_0 | n \rangle = E_n | n \rangle.$$  

(3)

At $t = 0$, it is known that only the lower level is populated; that is, $c_1(0) = 1$ and $c_2(0) = 0$, where

$$| \tilde{\psi}(t) \rangle = \sum_n | n \rangle \langle n | \tilde{\psi}(t) \rangle \equiv \sum_n c_n(t) | n \rangle.$$  

(4)

(a) Find $|c_1(t)|^2$ and $|c_2(t)|^2$ for $t > 0$ by solving for the quantum dynamics exactly.

(b) Solve the same problem but now using time-dependent perturbation theory to lowest non-vanishing order. Compare the two approaches for small values of $\gamma$. Treat the following two cases separately: (i) $\omega$ very different from $\omega_{12}$ and (ii) $\omega$ very close to $\omega_{12}$.

2. In class we gave a first-order formula for $P_{k\rightarrow s}(t)$, the probability for a system in eigenstate $|s\rangle$ to find itself in a different eigenstate of $H_0$ with energy $E_k$. Use this formula, which is valid for $k \neq s$, to calculate the probability that the system remains in state $s$. Show that the result agrees with $P_{s\rightarrow s}(t)$ calculated from $\langle s | \tilde{T}(t, t_0) | s \rangle$, only if if the second-order terms are retained in the perturbation expansion of this diagonal matrix element.