944 QUANTUM MECHANICS - Homework #4. Due: Friday, November 2, 2007.

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1. This problem is about Rabi's formula (see Sakurai, chapter 5 and problem 5.30). Consider a two-level system with $E_1 < E_2$. There is a time-dependent potential that connects the two levels as follows:

$$V_{11} \;=\; V_{22} \;=\; 0, \qquad V_{12} \;=\; \gamma\, e^{i\omega t}, \qquad V_{21} \;=\; \gamma\, e^{-i\omega t} \quad (\gamma \text{ real})\;, \tag{1}$$

where

$$V_{nm} \;\equiv\; \langle n|\, V\, |m\rangle \;, \tag{2}$$

and

$$H_0\, |n\rangle \;=\; E_n\, |n\rangle \;. \tag{3}$$

At $t = 0$, it is known that only the lower level is populated; that is, $c_1(0) = 1$ and $c_2(0) = 0$, where

$$\left|\tilde{\psi}(t)\right\rangle \;=\; \sum_n \,|n\rangle\, \langle n| \left|\tilde{\psi}(t)\right\rangle \;\equiv \sum_n \, c_n(t)\, |n\rangle \;. \tag{4}$$

   (a) Find $|c_1(t)|^2$ and $|c_2(t)|^2$ for $t > 0$ by solving for the quantum dynamics *exactly*.

   (b) Solve the same problem but now using time-dependent perturbation theory to lowest non-vanishing order. Compare the two approaches for small values of γ. Treat the following two cases separately: (i) ω very different from ω_{12} and (ii) ω very close to ω_{12}.

2. In class we gave a first-order formula for $P_{k\leftarrow s}(t)$, the probability for a system in eigenstate $|s\rangle$ to find itself in a different eigenstate of H_0 with energy E_k. Use this formula, which is valid for $k \neq s$, to calculate the probability that the system remains in state s. Show that the result agrees with $P_{s\leftarrow s}(t)$ calculated from $\langle s|\, \tilde{T}(t, t_0)\, |s\rangle$, only if if the second-order terms are retained in the perturbation expansion of this diagonal matrix element.