961 QUANTUM FIELD THEORY - Midterm - Due: 4/03/2007.

1. **Dilatations**. The theory of a free massless scalar field

$$\frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi \tag{1}$$

possesses a one-parameter family of symmetry transformations

$$\lambda: \quad \phi(x) \;\to e^\lambda\phi(e^\lambda x)\ . \tag{2}$$

These are called scale transformations, or dilatations. Compute the associated conserved current and conserved charge.

2. **Yukawa Theory**. Consider the Yukawa theory of interacting scalar and Dirac fields. Two Dirac fermions Ψ^i form a doublet of the isospin symmetry group $SU(2)$ while three real scalar fields Φ^a form an isotriplet. The Lagrangian is

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{2}\,\partial_\mu\Phi^a\,\partial^\mu\Phi^a \;-\; \frac{1}{2}m^2\,\Phi^a\Phi^a \;+\; \overline{\Psi}^{\bar\imath}(i\,\partial\!\!\!/ - M)\delta_{\bar\imath j}\Psi^j \\ & -\, g\,\Phi^a\,\overline{\Psi}^{\bar\imath}(\frac{1}{2}\tau^a)_{\bar\imath j}\Psi^j \;-\; \frac{1}{4!}\lambda\,(\Phi^a\Phi^a)^2 \end{aligned} \tag{3}$$

where τ^a are the Pauli matrices for the isospin (not to be confused with the σ spin matrices).

   (a) Write down the classical equation of motion for all the fields.

   (b) Verify the isospin symmetry of the Lagrangian, write down the isospin Noether currents J^a_μ and verify that they are conserved.

   (c) Write down the isospin charges $\hat{T}^a$ of the quantum field theory in terms of the bosonic and fermionic creation and annihilation operators and verify the isospin commutation relations: $[\hat{T}^a, \hat{T}^b] = i\epsilon^{abc}\hat{T}^c$.

3. **Gordon Identity**. Derive the *Gordon identity* for the Dirac spinors:

$$\bar{u}(p')\gamma^\mu u(p) \;=\; \frac{1}{2M}\,(p'+p)^\mu\,\bar{u}(p')u(p) \;+\; \frac{i}{2M}\,(p'-p)_\nu\,\bar{u}(p')\sigma^{\mu\nu}u(p)\,. \tag{4}$$