

1. Lorentz Group

Problem 3.1, Peskin and Schroeder

2. Dirac's γ Matrices

(a) Verify

$$[S^{\kappa\lambda}, S^{\mu\nu}] = i(g^{\lambda\mu}S^{\kappa\nu} - g^{\lambda\nu}S^{\kappa\mu} - g^{\kappa\mu}S^{\lambda\nu} + g^{\kappa\nu}S^{\lambda\mu}). \quad (1)$$

(b) Calculate

$$\{\gamma^\rho, \gamma^\lambda\gamma^\mu\gamma^\nu\}, \quad [\gamma^\rho, \gamma^\kappa\gamma^\lambda\gamma^\mu\gamma^\nu] \quad \text{and} \quad [S^{\rho\sigma}, \gamma^\lambda\gamma^\mu\gamma^\nu]. \quad (2)$$

(c) Show that

$$\begin{aligned} \gamma^\alpha\gamma_\alpha &= 4, \\ \gamma^\alpha\gamma^\nu\gamma_\alpha &= -2\gamma^\nu, \\ \gamma^\alpha\gamma^\mu\gamma^\nu\gamma_\alpha &= 4g^{\mu\nu}, \\ \gamma^\alpha\gamma^\lambda\gamma^\mu\gamma^\nu\gamma_\alpha &= -2\gamma^\nu\gamma^\mu\gamma^\lambda. \end{aligned} \quad (3)$$

(Hint: use $\gamma^\alpha\gamma^\nu = 2g^{\nu\alpha} - \gamma^\nu\gamma^\alpha$ repeatedly.)