

1. **Energy-Momentum Tensor, Noether's Theorem**

Problem 2.1, Peskin and Schroeder

2. **Quantization of Complex Scalar Fields**

Problem 2.2, Peskin and Schroeder

3. **Free Electromagnetic Fields***

Consider a pair of hermitian *vector* fields, namely the free electromagnetic fields $\hat{\mathbf{E}}(\mathbf{x}, t)$ and $\hat{\mathbf{B}}(\mathbf{x}, t)$. These are *transverse* vector fields

$$\nabla \cdot \hat{\mathbf{E}} = \nabla \cdot \hat{\mathbf{B}} = 0, \quad (1)$$

whose time-dependence

$$\begin{aligned} \frac{\partial \hat{\mathbf{B}}}{\partial t} &= -\nabla \times \hat{\mathbf{E}}, \\ \frac{\partial \hat{\mathbf{E}}}{\partial t} &= +\nabla \times \hat{\mathbf{B}}, \end{aligned} \quad (2)$$

follows from the free electromagnetic Hamiltonian

$$\hat{H}_{EM} = \int d^3\mathbf{x} \left(\frac{1}{2} \hat{\mathbf{E}}^2 + \frac{1}{2} \hat{\mathbf{B}}^2 \right). \quad (3)$$

Actually, the time-dependent Maxwell equations, eq. (2), follow from the Hamiltonian, eq.(3) *and the equal-time commutation relations:*

$$\begin{aligned} [\hat{E}_i(\mathbf{x}, t), \hat{E}_j(\mathbf{x}', t')] &= ???, \\ [\hat{B}_i(\mathbf{x}, t), \hat{B}_j(\mathbf{x}', t')] &= ???, \\ [\hat{E}_i(\mathbf{x}, t), \hat{B}_j(\mathbf{x}', t')] &= ??? \end{aligned} \quad (4)$$

Such commutation relations for the electromagnetic fields are completely determined by the consistency of eq. (2) with the Hamiltonian, eq. (3); **write them down**. Make sure your answer is consistent with the transversality of the fields, i.e., with the time-independent Maxwell's equations, eq. (1).