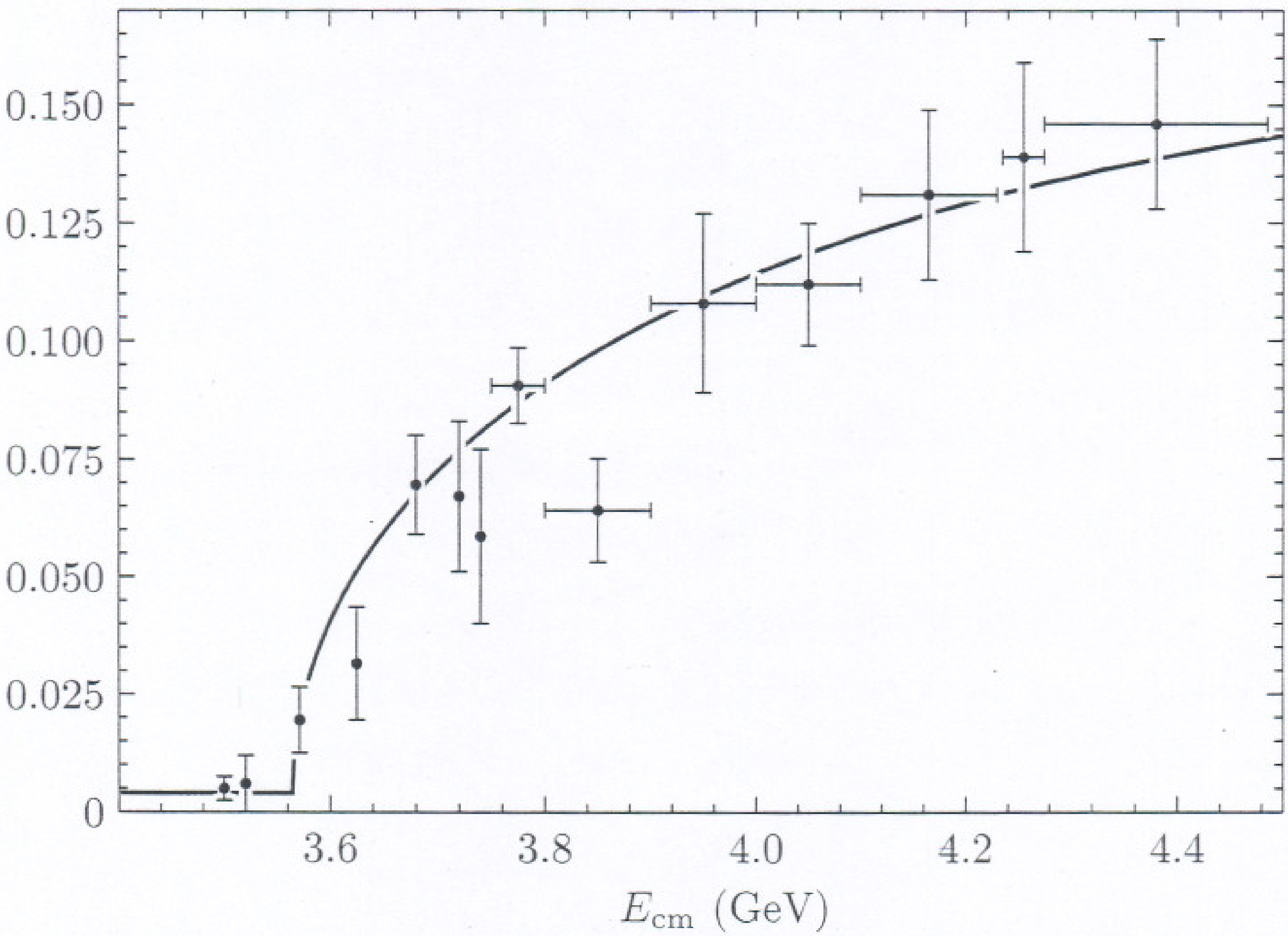
1



igure 5.2. The ratio $\sigma(e^+e^- \to \tau^+\tau^-)/\sigma(e^+e^- \to \mu^+\mu^-)$ of measured ross sections near the threshold for $\tau^+\tau^-$ pair-production, as measured the DELCO collaboration, W. Bacino, et. al., *Phys. Rev. Lett.* **41,** 13 978). Only a fraction of τ decays are included, hence the small overall ale. The curve shows a fit to the theoretical formula (5.13), with a small iergy-independent background added. The fit yields $m_\tau = 1782^{+2}_{-7}$ MeV.

$$\sigma_{tot} = \frac{8\pi\alpha^2}{3E_{c.m.}^2}\sqrt{1-\frac{m_\lambda^2}{E^2}}\left(1+\frac{1}{2}\frac{m_\lambda^2}{E^2}\right)$$

$e^+e^- \to$ hadrons

hadrons are strongly interacting particles, formed by Quantum Chromodynamics (QCD)

All observed hadrons are composed of quarks: q

2 structures: $\bar{q}q$ qqq
(pion) (proton)

q's come in various flavors: u d s c b t

+ each q comes in 3 colors

Simplest $e^+e^- \to$ hadrons process is:

$$\boxed{e^+e^- \longrightarrow q\bar{q}}$$

After $q\bar{q}$ pair is formed, q and $\bar{q}$ interact with other pairs to form pions, protons, etc.

Let's adapt $e^+e^- \to \mu^+\mu^-$ for use with $q\bar{q}$ final state.

3 modifications:

- Replace μ charge e with q charge $Q|e|$
- factor of 3 to count colours
- Include effect of interaction of $q\bar{q}$ pair.

	u	d	s	c	b	t
Q	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$

First 2 are easy!!

last is even easier: Asymptotic freedom

In high energy limit can neglect $q\bar{q}$ self interaction!!

Hence, expect

$$\boxed{\sigma(e^+e^- \to \mu^+\mu^-) \xrightarrow[E\to\infty]{} 3 \cdot Q^2 \frac{4\pi\alpha^2}{3E_{c.m}^2}}$$

conventional to define:

$$1 \text{ unit of } R \equiv \frac{4\pi\alpha^2}{3E_{c.m.}^2}$$

Hence σ is in units of $\sigma(e^+e^- \to \mu^+\mu^-)$

④

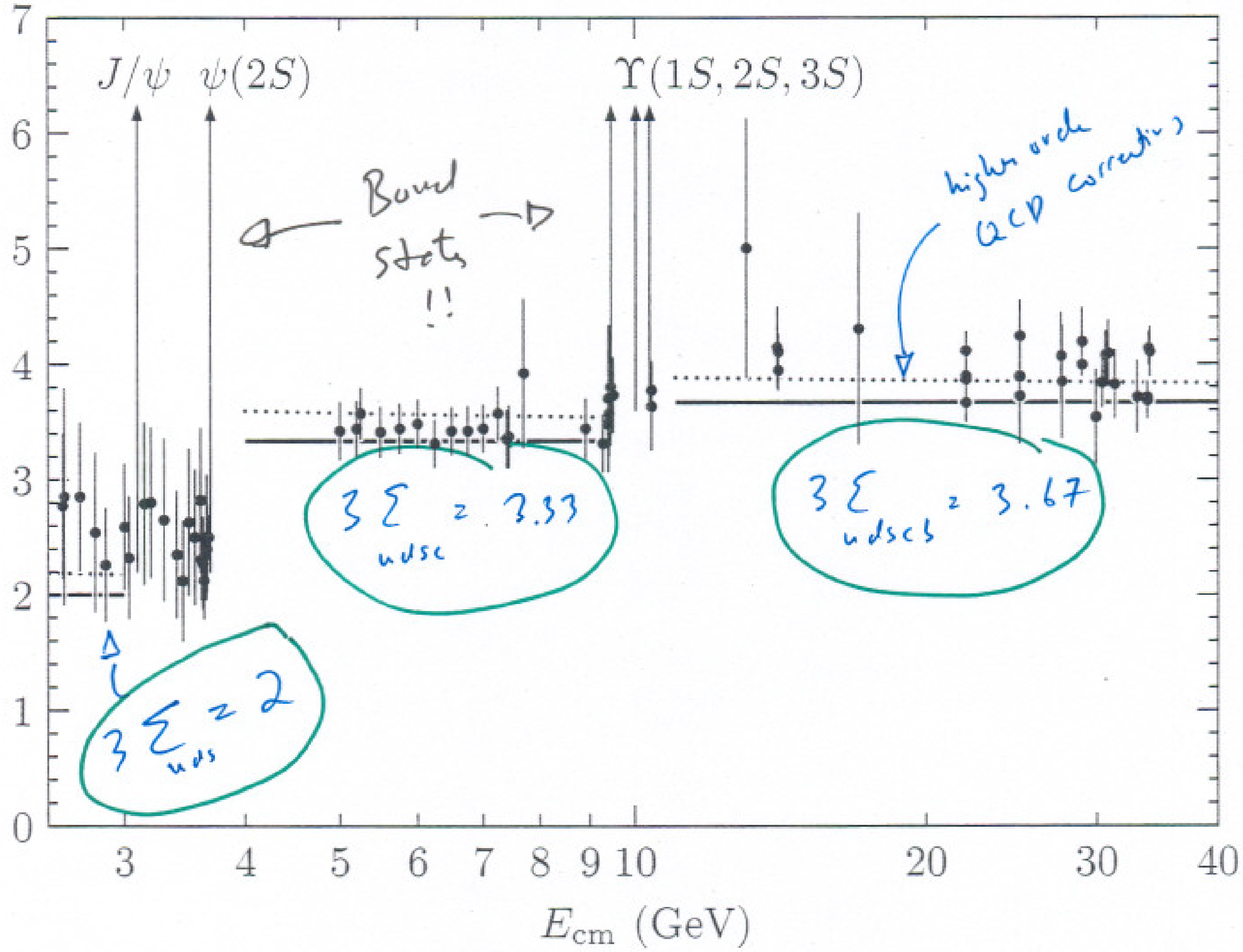
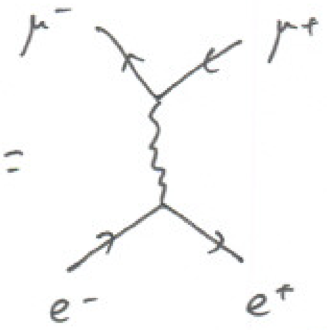


Figure 5.3. Experimental measurements of the total cross section for the reaction $e^+e^- \to$ hadrons, from the data compilation of M. Swartz, *Phys. Rev.* **D**53, 5268 (1996). Complete references to the various experiments are given there. The measurements are compared to theoretical predictions from Quantum Chromodynamics, as explained in the text. The solid line is the simple prediction (5.16).

$$\boxed{\frac{\sigma(\text{hadrons})}{\sigma(\mu^+\mu^-)} = 3 \cdot \left(\sum_i Q_i^2\right)}$$

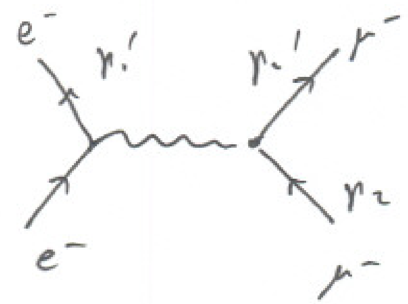
Recall: $e^- e^+ \to \mu^+ \mu^-$

$i\mathcal{M} =$ 

$= \dfrac{ie^2}{q^2}\, \bar{v}(p')\gamma^\mu u(p)\, \bar{u}(k)\gamma_\mu v(k')$

w/ $\boxed{\dfrac{1}{4}\sum |\mathcal{M}|^2 = \dfrac{e^4}{4q^4}\, \mathrm{tr}\left[(\not{p}' - m_e)\gamma^\mu(\not{p} + m_e)\gamma^\nu\right] \times \mathrm{tr}\left[(\not{k} + m_\mu)\gamma_\mu(\not{k}' - m_\mu)\gamma_\nu\right]}$

Now consider $e^- \mu^- \to e^- \mu^-$

$i\mathcal{M} =$ 

$= \dfrac{ie^2}{q^2}\, \bar{u}(p_1')\gamma^\mu u(p_1)\, \bar{u}(p_2')\gamma_\mu u(p_2)$

w/ $\boxed{\dfrac{1}{4}\sum |\mathcal{M}|^2 = \dfrac{e^4}{4q^4}\, \mathrm{tr}\left[(\not{p}_1' + m_e)\gamma^\mu(\not{p}_1 + m_e)\gamma^\nu\right] \times \mathrm{tr}\left[(\not{p}_2' + m_\mu)\gamma_\mu(\not{p}_2 + m_\mu)\gamma_\nu\right]}$

These are the same with replacement:

$$\boxed{p \to p_1 \qquad p' \to -p_1' \qquad k \to p_2' \qquad k' \to -p_2}$$

Relation between $e^+e^- \to \mu^+\mu^-$ and $e^-\mu^- \to e^-\mu^-$

is example of Crossing Symmetry

{ Equates S-matrix elements with interchange of particle with momentum p in initial state to antiparticle with momentum $k=-p$ in final state. }

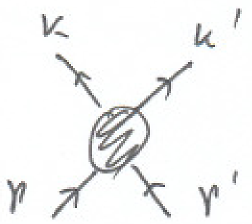
As require $p^0, k^0 > 0$ there is no p, k for which both processes are physical:

Amplitudes related through analytic continuation

There is nice set of variables that are useful for identifying crossing relations:

Mandelstam Variables

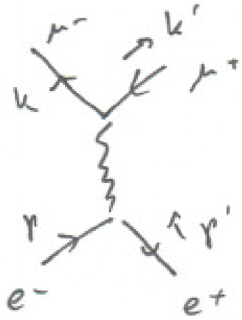
{ relevant for $2 \to 2$ scattering }



$$\boxed{\begin{aligned} s &= (p+p')^2 = (k+k')^2 \\ t &= (k-p)^2 = (k'-p')^2 \\ u &= (k'-p)^2 = (k-p')^2 \end{aligned}}$$

Let's illustrate utility of Mandelstam variables:

$e^+e^- \to \mu^+\mu^-$ (in massless limit)



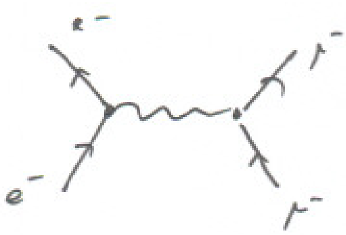
$$\frac{1}{4}\sum |M|^2 = \frac{8e^4}{q^4}\left[(p\cdot k)(p'\cdot k') + (p\cdot k')(p'\cdot k)\right]$$

$$(q = p + p')$$

$$\left\{\begin{aligned} s &= (p+p')^2 = q^2 \\ t &= (k-p)^2 = (k'-p')^2 = -2\,p\cdot k = -2\,p'\cdot k' \\ u &= (k'-p)^2 = (k-p')^2 = -2\,p\cdot k' = -2\,p'\cdot k \end{aligned}\right\}$$

$$\Rightarrow \boxed{\frac{1}{4}\sum |M|^2 = \frac{8e^4}{s^2}\left[\left(\frac{t}{2}\right)^2 + \left(\frac{u}{2}\right)^2\right]}$$

Similarly,



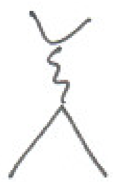
$$\boxed{\frac{1}{4}\sum |M|^2 = \frac{8e^4}{t^2}\left[\left(\frac{s}{2}\right)^2 + \left(\frac{u}{2}\right)^2\right]}$$

As we would expect, interchange of

t and s !!

But note that processes are very different physically.

Note:

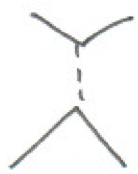
$\Big|$  $\Big|^2 \sim \frac{1}{s^2} \sim \frac{1}{(E_{c.m.}^2)^2}$

$\Big|$  $\Big|^2 \sim \frac{1}{t^2} \sim \frac{1}{(1-\cos\theta)^2} \sim \frac{1}{\theta^4}$ $(\theta \to 0)$

singular

Some terminology:

If there is one exchanged virtual particle:

s-channel  $\mathcal{M} \sim \frac{1}{s-m^2}$

t-channel  $\mathcal{M} \sim \frac{1}{t-m^2}$

u-channel  $\mathcal{M} \sim \frac{1}{u-m^2}$

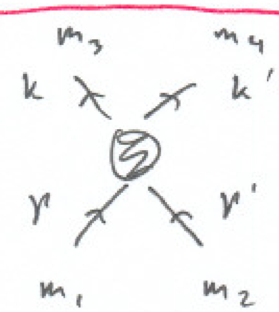
In c.m. frame we saw that $2 \to 2$ scattering is dependent on 2 independent variables:

$E_{c.m}$ and θ

But we have 3 mandelstam variables, $\underline{s\ t\ u}$.

So they $\underline{\text{cannot}}$ all be independent!

Consider general $2 \to 2$ process:



$$s = (p+p')^2 = (k+k')^2 = m_1^2 + m_2^2 + 2p\cdot p' = m_3^2 + m_4^2 + 2k\cdot k'$$

$$t = (k-p)^2 = (k'-p')^2 = m_3^2 + m_1^2 - 2k\cdot p = m_4^2 + m_2^2 - 2k'p'$$

$$u = (k'-p)^2 = (k-p')^2 = m_4^2 + m_1^2 - 2k'\cdot p = m_3^2 + m_2^2 - 2k\cdot p'$$

And note: $\underline{(p+p'-k-k')^2 = 0}$ (E-M conserv)

$$0 = m_1^2 + m_2^2 + m_3^2 + m_4^2 + 2p\cdot p' - 2p\cdot k - 2p\cdot k' - 2p'\cdot k - 2p'\cdot k' + 2k\cdot k'$$

$$2(s+t+u) = 2(m_1^2+m_2^2+m_3^2+m_4^2) + \underbrace{m_1^2+m_2^2+m_3^2+m_4^2+2p\cdot p' - 2p\cdot k - 2p\cdot k' - 2p'\cdot k - 2p'\cdot k' + 2k\cdot k'}_{0}$$

$$\Rightarrow \boxed{s+t+u = \sum_{i=1}^{4} m_i^2}$$