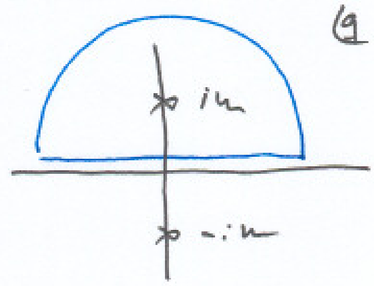
$$= -g^2 \frac{2\pi}{(2\pi)^3} \int_{-1}^{1} d\cos\theta \int_0^\infty dq\, q^2 \, \frac{e^{iqr\cos\theta}}{q^2 + m_\varphi^2}$$

$$= \frac{-g^2}{4\pi^2} \int_0^\infty dq\, q^2 \, \frac{(e^{iqr} - e^{-iqr})}{iqr} \, \frac{1}{q^2 + m_\varphi^2}$$

$$= \frac{-g^2}{4\pi^2 i r} \int_{-\infty}^{\infty} dq \, \frac{q\, e^{iqr}}{q^2 + m_\varphi^2}$$



$$\Rightarrow \boxed{V(r) = -\frac{g^2}{4\pi} \, \frac{1}{r} \, e^{-m_\varphi r}}$$

"ATTRACTIVE" Yukawa Potential

range is set by φ Compton wavelength: $\frac{1}{m_\varphi} = \frac{\hbar}{m_\varphi c}$

Relevant to theory of Nuclear forces

$\psi \to$ Nucleons

$\varphi \to$ pions

Yukawa used observationally determined range to estimate pion mass $\sim$ 150 MeV

Quantum Electrodynamics (Q.E.D.)

Recall the Lagrangian:

$$\boxed{\mathcal{L}_{QED} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}}$$

$D_\mu \equiv \partial_\mu + ie A_\mu$

$e = -|e|$ (charge of electron)

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Q.E.D. based on:

- U(1) gauge symmetry: $\psi \to e^{i\alpha(x)}\psi$

  $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \alpha(x)$

- Renormalizability:

  no operators with dimension > 4.

$$\boxed{H_{int} = \int d^3x \quad e\bar{\psi}\gamma^\mu\psi A_\mu}$$

Very similar to Yukawa theory!!

But has complicating issues:

- photon A_μ is massless $\Rightarrow$ infrared issues
- gauge invariance = redundancy $\Rightarrow$

complicates derivation of Feynman rules

One way to think of how problems arise:

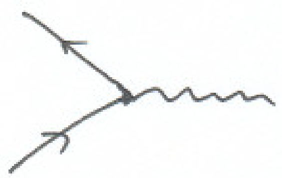
$\dot{A}_0$ does not appear in $\mathcal{L}$.

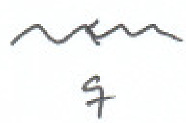
Hence "momentum" conjugate to A_0 is not there.

Contradicts $[A^0(x), \pi^0(y)] = i\delta^{(3)}(x_i - y_i)$

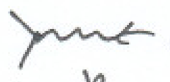
Path integral formulation of Q.F.T. offers elegant solution.

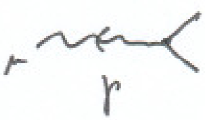
For now, guess Feynman Rules:

 $\mu \quad = -ie\gamma^\mu$

μ  ν, q $\quad = \dfrac{-i g_{\mu\nu}}{q^2 + i\epsilon}$

External photons:

$\overbrace{A_\mu | p_i\rangle}$ =  μ, p $= \epsilon_\mu(p)$

$\overbrace{\langle p_i | A_\mu}$ = μ  p $= \epsilon^*_\mu(p)$

polarization vectors

Work in Lorenz gauge to maintain manifest Lorentz invariance: (not Lorentz)

$$\boxed{\partial_\mu A^\mu = 0}$$

Recall field equation for A_μ:

$$\boxed{\partial_\mu F^{\mu\nu} = 0}$$

$$\Rightarrow \partial_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu) = 0$$

$$\Rightarrow \Box A^\nu - \partial^\nu (\partial_\mu A^\mu) = 0$$

So in Lorenz gauge:

$$\boxed{\Box A_\mu = 0}$$

Hence, each component of A_μ satisfies KG equation!!

Momentum space solutions are of form:

$$\underbrace{\epsilon_\mu(p)}_{\text{4-vector}} e^{-ip\cdot x} \qquad \underline{p^2 = 0}$$

what about quantized E&M field??

$$\boxed{A_\mu(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{p_i}}} \sum_{r=0}^{3} \left(a^r_{p_i} \epsilon^r_\mu(p) e^{-ip\cdot x} + a^{r\dagger}_{p_i} \epsilon^{r*}_\mu(p) e^{ip\cdot x} \right)}$$

Basis of polarization vectors

$$\epsilon^\mu = (0, \epsilon_i) \qquad p^\mu = (|p_i|, p_i)$$

restricted to transverse polarization

Gauge condition: $\boxed{\epsilon^\mu p_\mu = 0}$

$\Rightarrow \quad \epsilon_i p_i = 0$ transverse polarization

If p_i points along z,

$$\boxed{\epsilon^\mu = (0, 1, \pm i, 0)\frac{1}{\sqrt{2}} \quad \begin{matrix}\text{right-}\\ \text{left-}\end{matrix}\ \text{handed}}$$

Why $-g_{\mu\nu}$ in photon propagator ??

(Problem: because $g_{\mu\nu}$ is not positive definite A_0 creates states of negative norm)

Consider how problem arises:

So $\mathcal{L} \to \mathcal{L} - \frac{1}{2}(\partial_\mu A^\mu)^2$ (adds $\dot{A}^0$ term)

$$[A^\mu(x_i), \dot{A}^\nu(y_i)] = -i g^{\mu\nu} \delta^{(3)}(x_i - y_i)$$

$[A^0(x_i), \dot{A}^0(y_i)) \Rightarrow$ negative norm states created by $A^0 \sim a^{0\,+}_{p_i}$

That is, $\boxed{[a^0_{p_i}, a^{0\,+}_{k_i}] = -2k_0 (2\pi)^3 \delta^{(3)}(k_i - p_i)}$

So $|\chi\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k_0} f(k)\, a^{0\,+}_{k_i} |0\rangle$

$$\therefore \quad \langle \chi | \chi \rangle = - \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k_0} |f(k)|^2 \langle 0|0 \rangle$$

OUCH! negative norm

But one can show that these states do not contribute to physical processes!!

Must await path integral formulation to give careful derivation of photon propagator.