I. β DECAY

1. The differential decay rate for the β^--decay reaction $A(Z,N) \rightarrow A(Z+1,N-1) + e^- + \bar{\nu}_e$ can be written as

$$d\mathcal{W} \;=\; \frac{c^6}{8\pi^5\hbar}\left(\frac{2.4G_F}{(\hbar c)^3}\right)^2 |M|^2\delta(\Delta B - E_\nu - E_e)d^3p_e d^3p_v \;. \qquad (1)$$

Here ΔB is the difference in the binding energies of parent and daughter nucleus. The factor 2.4 takes into account the relative coupling factors for Fermi and Gamow-Teller type transitions. The nuclear matrix element M satisfies the condition: $|M|^2 \leq 1$.

(a) Integrate over the unobserved neutrino momentum to obtain the differential rate as a function of the electron energy. Assume that neutrinos are massless.

(b) Next integrate over the electron momentum in order to obtain the total decay rate. How does the rate scale with $Q \equiv \Delta B - m_e c^2$ in the limit $Q \gg m_e c^2$?

(c) Estimate the total decay rate for $|M|^2 = 1$ and $Q = 1$ MeV as well as $Q = 10$ MeV.