720/820 Nuclear Physics - Homework #3. Due: Monday, 3/22/10

I. THE LEVI-CIVITA TENSOR

1. Prove the following vector identities by using the properties of the Kronecker delta and the Levi-Civita tensor.

   (a)
$$\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \vec{B}\left(\vec{A} \cdot \vec{C}\right) - \vec{C}\left(\vec{A} \cdot \vec{B}\right) ; \qquad (1)$$

   (b)
$$\left(\vec{A} \times \vec{B}\right) \cdot \left(\vec{C} \times \vec{D}\right) = \left(\vec{A} \cdot \vec{C}\right)\left(\vec{B} \cdot \vec{D}\right) - \left(\vec{A} \cdot \vec{D}\right)\left(\vec{B} \cdot \vec{C}\right) . \qquad (2)$$

II. THE ELECTROMAGNETIC MULTIPOLE EXPANSION

2. As in class, consider a charge distribution $\rho(x)$ that is non-vanishing in some region of space and is interacting with an electromagnetic potential that is slowly varying over the volume of the charge distribution. The interaction Hamiltonian is

$$H_{int} = \int d^3x \, j_\mu(x) A^\mu(x) . \qquad (3)$$

   (a) Show that one can write

$$H_{int} = Q\phi(0) - \mathbf{P} \cdot \mathbf{E} - \frac{1}{6} Q^{ij} \partial^i E^j - \mathbf{M} \cdot \mathbf{B} + \dots , \qquad (4)$$

   where the moments $\mathbf{P}$, $\mathbf{M}$ and Q^{ij} were defined in class.

   (b) Discuss the significance of the ... in this expansion. What is this an expansion in?

   (c) Calculate $\mathbf{P}$ and Q^{ij} if the surface of a deformed nucleus is given by the equation

$$x^2 + y^2 + \mu z^2 = R^2 , \qquad (5)$$

   where $R = \mu A^{1/3}$ fm. Assume that $\mu = 1$, $A = 300$ and $Z = 80$, and the nuclear density is uniform inside the surface and zero outside.

3. *Wong*, Problem 4.7

4. *Wong*, Problem 4.12

5. *Wong*, Problem 4.13

6. *Wong*, Problem 4.15

7. *Wong*, Problem 4.19