720/820 Nuclear Physics - Homework #2. Due: Friday, 2/26/10

I. THE CROSS SECTION

1. The total cross-section, σ, is related to the probability that scattering occurs, and, as shown in class, is given by the formula

$$\sigma(p) \;=\; \int d\Omega\, |f_p(\Omega)|^2 \;, \tag{1}$$

where f_p is the scattering amplitude. As the cross-section carries units of distance-squared, one can visualize it as the effective cross-sectional area of the target particles "seen" by the beam particles. A more precise definition is

$$\sigma \;=\; \frac{\textit{number of reactions per unit time}}{\textit{beam particles per unit time} \times \textit{scattering centers per unit area}} \;. \tag{2}$$

(a) Use eq. 2 to argue that the differential cross-section can be written as

$$\frac{d\sigma}{d\Omega} \;=\; r^2\, \frac{|\mathbf{J}_{scattered}|}{|\mathbf{J}_{incident}|} \;, \tag{3}$$

where r is the radial coordinate and $\mathbf{J}$ is the probability current.

(b) Use the asymptotic form of the total wavefunction, ψ^+, written in terms of an incoming plane wave and an outgoing spherical wave, together with eq. 3, to recover eq. 1.

II. THE BORN APPROXIMATION

2. For a central potential, $V(r)$, the Born approximation for the scattering amplitude may be written as

$$f_q(\Omega) \;=\; -\frac{m}{2\pi} \int d^3r' V(r') e^{-i\vec{q}\cdot\vec{r}'} \tag{4}$$

where $\vec{q} = \vec{p}\,' - \vec{p}$ is the "momentum transfer", and $\vec{p}$ and $\vec{p}\,'$ are the momenta of a particle (e.g. an α-particle) before and after scattering, respectively. This particle scatters off a target (e.g. gold foil) at rest in the lab frame.

(a) Draw a picture of the scattering process described above and label all momenta and the scattering angle, θ.

(b) Using energy conservation, find the relation between the magnitude of $\vec{q}$, the magnitude of $\vec{p}$, and the scattering angle θ.

(c) Evaluate the angular part of the integral in eq. 4.

(d) Now assume a potential of the form

$$V(r) \;=\; V_0 \,\frac{e^{-\alpha r}}{\alpha r} \;. \tag{5}$$

Here $1/\alpha$ may be viewed as the "range" of the potential. For instance, with $\alpha = 0$, this is the Coulomb potential, mediated by photon exchange.

Evaluate the scattering amplitude with arbitrary α.

(e) Finally, calculate the differential cross section, and discuss the $\alpha \to 0$ limit. This is the Rutherford cross section. (HINT: note that the ratio V_0/α must be finite as $\alpha \to 0$.)

III. TOY MODEL OF THE NN SYSTEM

3. Here we will view the NN force as being due to the exchange of a spinless particle with mass equal to that of the pion,

$$V(r) \;=\; -\beta\,\frac{e^{-m_\pi r}}{r} \;. \tag{6}$$

We will choose $\beta = 0.070$. Consider a single nucleon incident upon a single nucleon at rest in the lab frame. Denote the incident nucleon kinetic energy as T_{lab}. Assume that the nucleons are interacting in the channel with total spin zero. (This problem will require numerical work.)

(a) Plot the s-wave radial wavefunction $u_0(r)$ as a function of r for this potential, and on the same plot show the corresponding radial wavefunction $u_0(r)$ with the potential turned off.

(b) Determine the s-wave scattering length.

(c) Determine the s-wave phase shift and plot it in degrees vs. T_{lab} between 0 and 100 MeV.

(d) Compare with experimental data available at: http://nn-online.sci.kun.nl/NN/NNonline.html , and comment on how good the agreement is.