1. Goldstein Derivation 1.9
2. Goldstein Exercise 1.15
3. Goldstein Exercise 1.21
4. Fetter Problem 3.3
5. Fetter Problem 3.6

6. Fermat’s principle is the statement that a ray of light in a medium with a variable index of refraction will follow the path that requires the shortest travel time. For two-dimensional travel in the $x – y$ plane, show that such a path is obtained by minimizing the quantity:

$$cT = \int_{a}^{b} n(x, y) \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx.$$  

(Note: in a medium with index of refraction $n$, the speed of light is $c/n$.)

For the particular case $n = \kappa y$ for $y > 0$, show that the rays of light travel along paths of the form

$$(x - x_0) + y^2 = a^2,$$  

which are semi-circles centered upon the x-axis.

7. A uniform string of length $\sigma$ is draped over the edge of a horizontal table, initially at rest with a piece of length $x_0$ extending over the edge. Show that the amount of string hanging over the edge at time $t$, $x(t)$, satisfies

$$\sigma \ddot{x} = g x,$$  

so that

$$x(t) = x_0 \cosh \sqrt{\frac{g}{\sigma}} t.$$  

8. Show that, for a planar curve $y(x)$ connecting two points, $(a, y(a))$ and $(b, y(b))$, with a fixed length

$$\ell = \int_{a}^{b} \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx$$  

the minimum area under the curve,

$$A = \int_{a}^{b} y(x) dx$$  

occurs for a circular arc passing through these points.

Show that the ratio of the Euclidean distance between points $d$ and the length $\ell$ of the curve are related to the parameter $\rho$ through the trancendental equation

$$\frac{d}{\ell} = \frac{\sin \rho}{\rho}.$$  

where the radius $R$ of the arc is

$$R = \frac{\ell}{2\rho}. \quad (8)$$

There is one solution $\rho$ of eq. 7 on the interval $0 \leq \rho \leq \pi$, since $d < \ell$ and the function on the right-hand side is monotonically decreasing in $\rho$. The radius $R$ may then be determined. Show that the area under the curve is

$$A = \frac{1}{2} (y(a) + y(b))(b - a) + R^2 (\rho - \sin \rho \cos \rho). \quad (9)$$