939 CLASSICAL MECHANICS - Homework #2. Due: Thursday, 10/1/2009

---

1. Goldstein Derivation 1.9
2. Goldstein Exercise 1.15
3. Goldstein Exercise 1.21
4. Fetter Problem 3.3
5. Fetter Problem 3.6
6. Fermat's principle is the statement that a ray of light in a medium with a variable index of refraction will follow the path that requires the shortest travel time. For two-dimensional travel in the $x-y$ plane, show that such a path is obtained by minimizing the quantity:

$$cT = \int_a^b n(x,y)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \; . \tag{1}$$

   (Note: in a medium with index of refraction n, the speed of light is c/n.)

   For the particular case $n = \kappa y$ for $y > 0$, show that the rays of light travel along paths of the form

$$(x - x_0) \; + \; y^2 \; = \; a^2 \; , \tag{2}$$

   which are semi-circles centered upon the x-axis.

7. A uniform string of length σ is draped over the edge of a horizontal table, initially at rest with a piece of length x_0 extending over the edge. Show that the amount of string hanging over the edge at time t, $x(t)$, satisfies

$$\sigma \ddot{x} \; = \; gx \; , \tag{3}$$

   so that

$$x(t) \; = \; x_0 \cosh \sqrt{\frac{g}{\sigma}} t \; . \tag{4}$$

8. Show that, for a planar curve $y(x)$ connecting two points, $(a, y(a))$ and $(b, y(b))$, with a fixed length

$$\ell \; = \; \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \tag{5}$$

   the minimum area under the curve,

$$A \; = \; \int_a^b y(x)dx \tag{6}$$

   occurs for a circular arc passing through these points.

   Show that the ratio of the Euclidean distance between points d and the length ℓ of the curve are related to the parameter ρ through the trancendental equation

$$\frac{d}{\ell} \; = \; \frac{\sin\rho}{\rho} \tag{7}$$

where the radius R of the arc is

$$R = \frac{\ell}{2\rho} . \tag{8}$$

There is one solution ρ of eq. 7 on the interval $0 \leq \rho \leq \pi$, since $d < \ell$ and the function on the right-hand side is monotonically decreasing in ρ. The radius R may then be determined. Show that the area under the curve is

$$A = \frac{1}{2}(y(a) + y(b))(b - a) + R^2(\rho - \sin\rho\cos\rho) . \tag{9}$$