939 CLASSICAL MECHANICS - Homework #1. Due: Thursday, 9/17/2009

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1. Prove the following vector identities by using the properties of the Kronecker delta and the Levi-Civita tensor.

   (a)
   $$\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \vec{B}\left(\vec{A} \cdot \vec{C}\right) - \vec{C}\left(\vec{A} \cdot \vec{B}\right) ; \tag{1}$$

   (b)
   $$\left(\vec{A} \times \vec{B}\right) \cdot \left(\vec{C} \times \vec{D}\right) = \left(\vec{A} \cdot \vec{C}\right)\left(\vec{B} \cdot \vec{D}\right) - \left(\vec{A} \cdot \vec{D}\right)\left(\vec{B} \cdot \vec{C}\right) . \tag{2}$$

2. In class we showed that the free particle has three constants of the motion given by momentum; i.e. $\vec{\Gamma}_1 = \vec{p}$. Does the free particle have other constants of the motion that are not implied by conservation of momentum? If so, what are they and what do they represent physically?

3. A point mass m slides without friction inside a surface of revolution $z = \alpha \sin(r/R)$ whose symmetry axis lies along the direction of a uniform gravitational field $\vec{g}$. Here α and R are constants with dimensions of length. Consider $0 < r/R < \pi/2$.

   (a) Construct the Lagrangian $\mathcal{L}\left(r, \phi, \dot{r}, \dot{\phi}\right)$ and compute the equations of motion for r and ϕ.

   (b) Are there stationary horizontal circular orbits?

   (c) Which of these orbits is stable under small impulses along the surface transverse to the direction of motion?

   (d) If the orbit is stable, what is the frequency of oscillation about the equilibrium point?

4. Goldstein Derivation 1.1

5. Goldstein Derivation 1.4

6. Goldstein Exercise 1.13

7. Goldstein Exercise 1.14