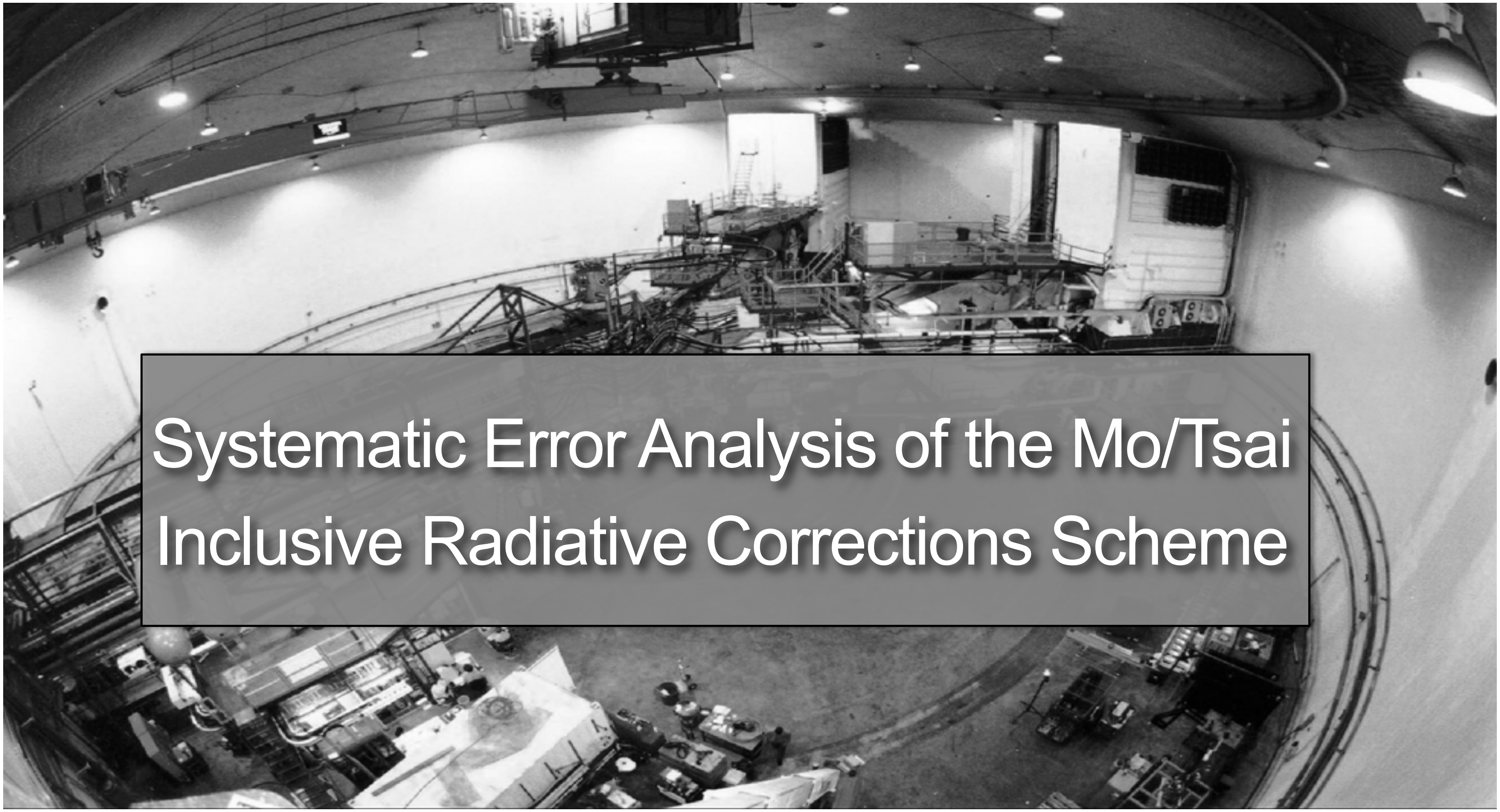

Systematic Error Analysis of the Mo/Tsai Inclusive Radiative Corrections Scheme

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Jefferson Lab – Radiative Corrections Workshop

May 16-19, 2016

University of New Hampshire

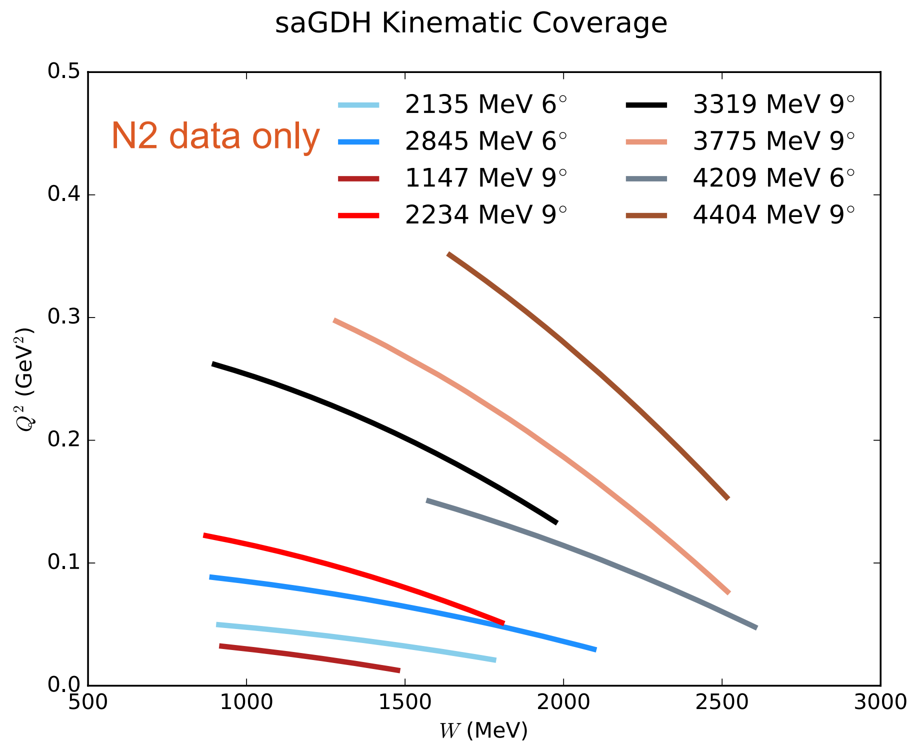
TALK OUTLINE

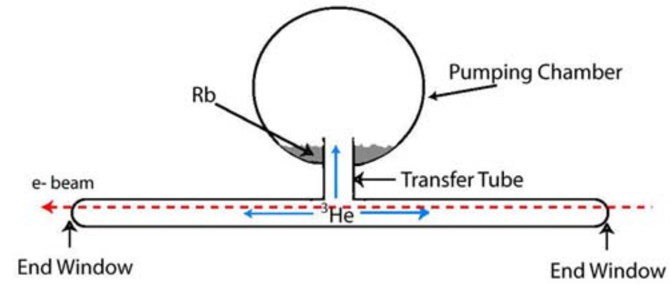
- Overview of the Small Angle GDH experiment
- Overview of the Mo and Tsai radiative corrections scheme
  - Elastic tail subtraction systematic error
  - Inelastic radiative corrections systematic error
- Results for the Small Angle GDH experiment nitrogen data set

SMALL ANGLE GDH

- Jefferson Lab Hall A experiment
  - Polarized He3 gas target
  - Nitrogen data set as well
  - Inclusive measurement
- Low Q^2 data (0.02 to 0.35 GeV^2)
- Systematic error analysis focused on low Q^2 nitrogen data set
  - Data set split among 6°/9° scattering and also partial/full W coverage

$$Q^2 = -q^2 = 4E_s E_p \sin^2\frac{\theta}{2}$$
$$W^2 = M_p^2 + 2M_p\nu - Q^2$$
$$\nu = E_s - E_p$$





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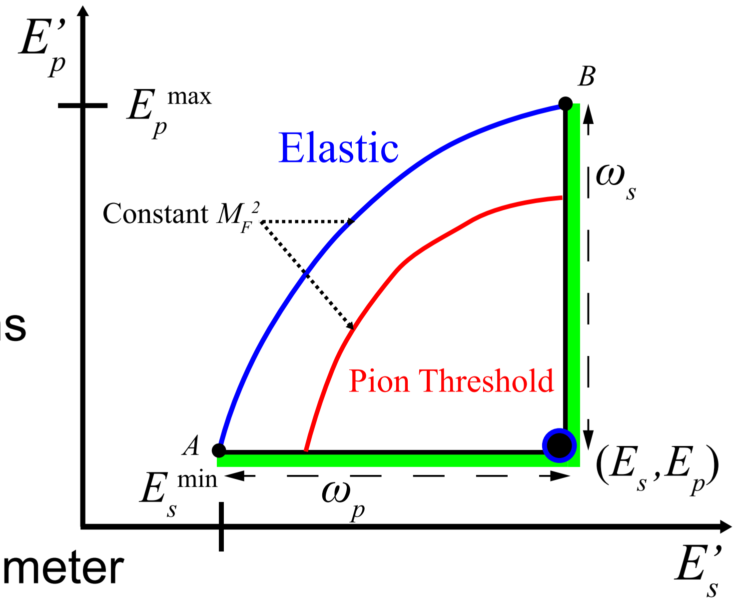
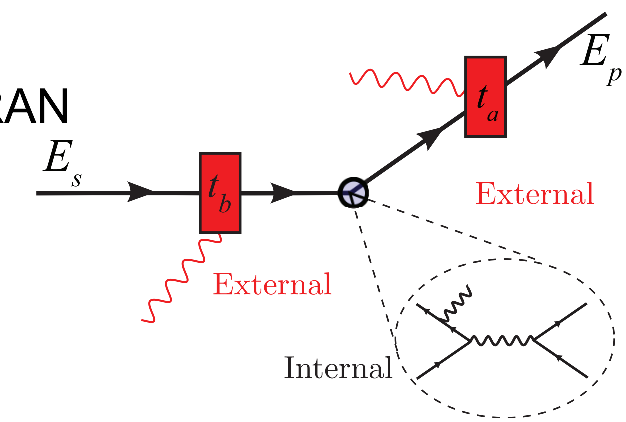
MO/TSAI RADIATIVE CORRECTIONS

‘Classic’ radiative corrections scheme

- L.W. Mo and Y.S. Tsai, Rev. Mod. Phys 41, 205 (1969)
- Y.S. Tsai, SLAC-PUB-848 (1971)



- Target structure accounted via structure functions
  - Scheme is model independent
- Mathematic formulation is non-covariant
- Diff between hard/soft photons done with *Δ* parameter
  - Results should be independent of parameter
- Systematic error analysis applies to set of FORTRAN codes: ROSETAIL and RADCOR
  - *Your mileage may vary*



ELASTIC TAIL SUBTRACTION

Elastic tail is accounting of all possible ways electron can lose energy and then scatter elastically into the detectors. Calculation includes:

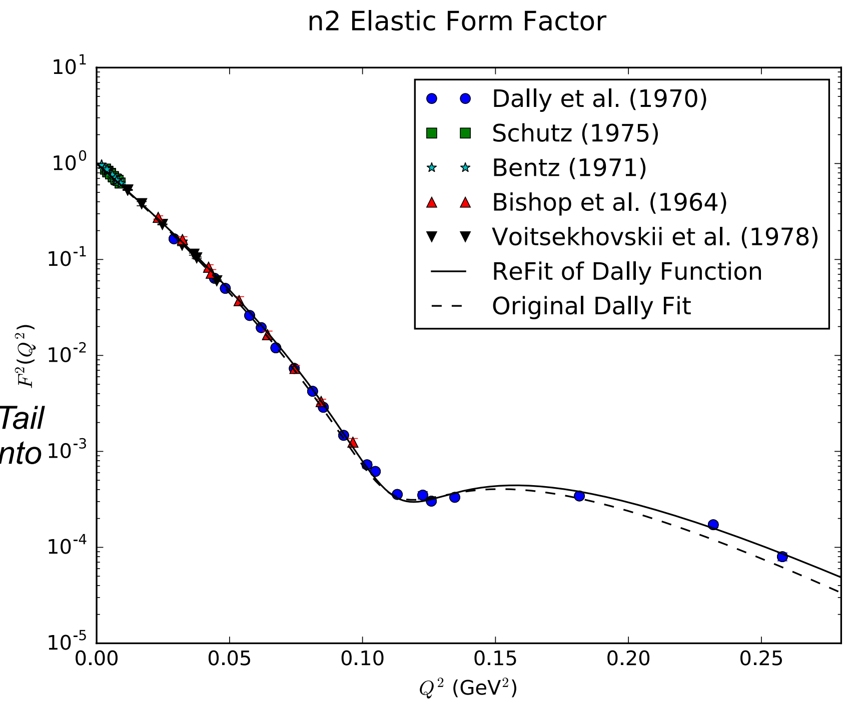
- Elastic Form Factors
    - FF's are calculated in first Born approximation (only single photon exchange)
    - Correction factor applied to take into account higher order virtual photon diagrams
    - Ignore anything happening at the hadron vertex
- Bremmstrahlung (emission of real photons)
    - Internal bremm. – occurs within the Coulombic field of the target nucleus
    - External bremm. – occurs within the Coulombic field of anything but the target
        - Technically not bremm. but external tail also includes collisional/ionizational energy loss (colliding with atomic electrons)
- Multiple photon corrections
    - Mo/Tsai assume single bremm. photon is emitted but in reality an infinite number of photons share the energy of that one photon
    - Mo/Tsai apply correction to both internal/external corrections to account for this

FORM FACTORS AND THE BORN APPROXIMATION

- Nitrogen form factor fit from E.B. Dally *et al.*, Phys. Rev. C 2 2057 (1970)
  - Additional data found to check the fit and fill in low Q^2 portion

Systematic uncertainty to first Born approximation assumption?

- Should be small because nitrogen is a light nucleus and saGDH is at low Q^2



- Source papers:
  - P. Barraeau *et al.*, "Deep-Inelastic Scattering from Carbon." Nucl. Phys. A402 515-540 (1983)
  - E. Borie, *"Correction to the Formula for the Radiative Tail in Elastic Electron Scattering." Lettere Al Nuovo Cimento 1 106 (1971)*
- They estimate the contribution as:

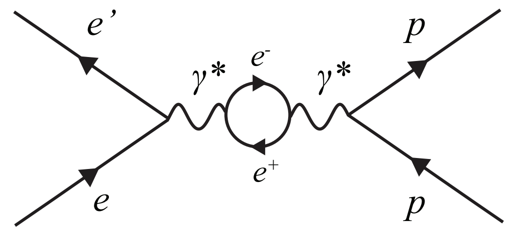
$$\delta = \pi\alpha Z \sin\frac{\theta}{2} / (1 + \sin\frac{\theta}{2})$$

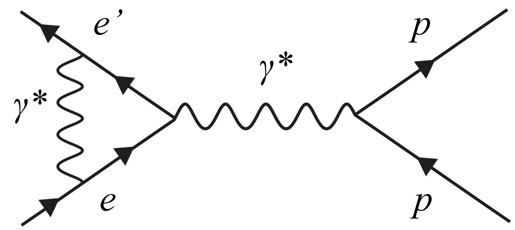
Results in systematic error for $Z = 7$ and $\theta = 6.0$ of δ~0.8%

MT HIGHER ORDER LOOPS

Mo/Tsai give their virtual photon corrections as sum of (call it $\tilde{F}$) :

- Vacuum: $$\delta_{\text{vac}} = \frac{2\alpha}{\pi}(-5/9 + \frac{1}{3}ln(Q^2/m_e^2))$$
- Vertex (non-infrared divergent): $$\delta_{\text{vertex}} = \frac{2\alpha}{\pi}(-1 + \frac{3}{4}ln(Q^2/m_e^2))$$
- Schwinger (soft-photon/non-infrared divergent): $$\delta_{\text{S}} = \frac{\alpha}{\pi}(\frac{\pi^2}{6} - \Phi(\cos^2\frac{\theta}{2})$$





- There is also a normalization term for the external bremsstrahlung:
  - Given as $1/\Gamma(1+bt)$ where b=4/3 and t is the sum of the radiation lengths
- Obviously could have vacuum loops according to muon and tau leptons
- Could also have quark loops in the vacuum diagram
- And also hadron vertex photon emission!

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AN UPDATED 'FBAR'

A full formula (not in limit $Q^2 >> m_e^2$) for vacuum contribution is given in a variety of references:

- B. Badalek, D.Bardin, K. Kurek and C. Scholz, "Radiative Corrections Schemes in Deep Inelastic Muon Scattering" arxiv:hep-ph/940328v1 (1994).
- S. Dasu *et al.*, "Measurement of Kinematic and Nuclear Dependence of R = SL/ST in Deep Inelastic Electron Scattering" Phys. Rev. D49 5641 (1994).

$$\delta_{\rm vac}^{e,\mu,\tau} = \frac{2\alpha}{\pi}\left[\frac{-5}{9} + \frac{4m_l^2}{3Q^2} + \frac{1}{3}\sqrt{1+\frac{4m_l^2}{Q^2}}\left(1-\frac{2m_l^2}{Q^2}\right)\ln\left(\frac{\sqrt{1+4m_l^2/Q^2}+1}{\sqrt{1+4m_l^2/Q^22}+1}\right)\right]$$

- Ignoring quark loops because they're sensitive to the quark mass and all parameterizations I found of this effect are at Q^2 > 1 GeV^2
  - Ignoring γ – Z interference terms as well
- Comparison between the M/T and updated vacuum diagram 'FBar' (1 + δ):
  - MT: Q^2 = 0.05 GeV^2 -> 'FBar' = 1.0577 && Q^2 = 0.02 GeV^2 -> 'FBar' = 1.053
  - Updated: Q^2 = 0.05 GeV^2 -> 'FBar' = 1.0625 && Q^2 = 0.02 GeV^2 -> 'FBar' = 1.056

Systematic for 'Fbar' term is δ~0.4%.

TARGET RADIATION

MT neglect any kind of target radiation for the internal elastic tail

- Sources for including this effect:
  - G. Miller *et al*. "Inelastic Electron-Proton Scattering at Large Momentum Transfers and the Inelastic Structure Functions of the Proton." Phys. Rev. D 5 528 (1972).
  - Guthrie Miller Ph.D thesis: "Inelastic Electron Scattering at Large Angles" Stanford 1971.
  - S. Stein *et al.* "Electron Scattering at 4° with Energies of 4.5-20 GeV", Phys. Rev. D 12 1884 (1975)

$$R_t = 1 + \frac{\alpha}{\pi b t_r}\left[\left(\frac{1+2\tau}{2\tau\sqrt{1+1/\tau}}\right)\ln(1+2\tau+2\tau\sqrt{1+1/\tau}) - 1 + 2\ln(\eta_s)\right]$$

$$bt_r = \frac{2\alpha}{\pi}(\ln(Q^2/m_e^2) - 1)$$

$$\tau = Q^2/4M_T^2$$

$$\eta_s = 1 + 2\frac{E}{M_T}\sin^2\frac{\theta}{2}$$

- At ν = 1200 MeV (E = 2135 Mev/θ = 6°):
  - R_t = 1.00019 so the systematic is negligible for nitrogen scattering at saGDH kinematics

MULTIPLE SOFT-PHOTON CORRECTION

Multiple photon correction applied to both internal and external tail

- Correction given by MT and Stein in the following papers
  - Y.S. Tsai, "Radiative Corrections to Electron Scattering." SLAC-PUB-848 1971
  - S. Stein *et al.,*"Electron Scattering at 4° with energies of 4.5-20 GeV." Phys. Rev. D 12 1884 (1975).
- MP is sizable correction (at saGDH kinematics) to tail ranging in value from ~0.60 to ~0.90 as you go from low-*v* to high-*v*
  - Especially important where tail is large!
- **Quoting Tsai: "There is some uncertainty in the validity of the [MP] factor. We know that this factor is correct when w_s/E_s and $w_p/(E_p+w_p)$ are small."**

$$\delta_{MP} = \left(\frac{\omega_s}{E_s}\right)^{(b(t_a+t_b)+bt_r/2)} \left(\frac{\omega_p}{E_p+\omega_p}\right)^{(b(t_a+t_b)+bt_r/2)}$$

$$\omega_s = E_s - \frac{E_p}{1-2E_p/M_T \sin^2\frac{\theta}{2}}$$

$$\omega_p = \frac{E_s}{1-2E_s/M_T \sin^2\frac{\theta}{2}} - E_p$$

$$bt_r = 2\alpha/\pi[\ln(Q^2/m^2)-1]$$

Energy of incoming (outgoing) photon

G. MILLER MULTIPLE SOFT-PHOTON CORRECTION

Guthrie Miller's thesis offers an alternative MP correction term

- Assuming only external effects Miller gives exact external bremm. tail as (this includes multiple photon processes)

Exact Tail

$$\frac{d\sigma}{d\Omega dE'} = \int_{E-\omega}^{E} \pi(E, E_1, t_b) \frac{d\sigma}{d\Omega}(E_1, \theta) \pi(E_1', E', t_a) dE_1$$

New MP factor

$$= \left(\frac{k}{\sqrt{EE'}}\right)^{bt_a+bt_b} \frac{1}{\Gamma(1+bt_b+bt_a)}$$

$$\times \left[t_b w(E, E-\omega)\eta'^2 \frac{d\sigma}{d\Omega}(E-\omega, \theta)\right.$$

$$\left. + t_a w(E'+\omega, E') \frac{d\sigma}{d\Omega}(E, \theta)\right]$$

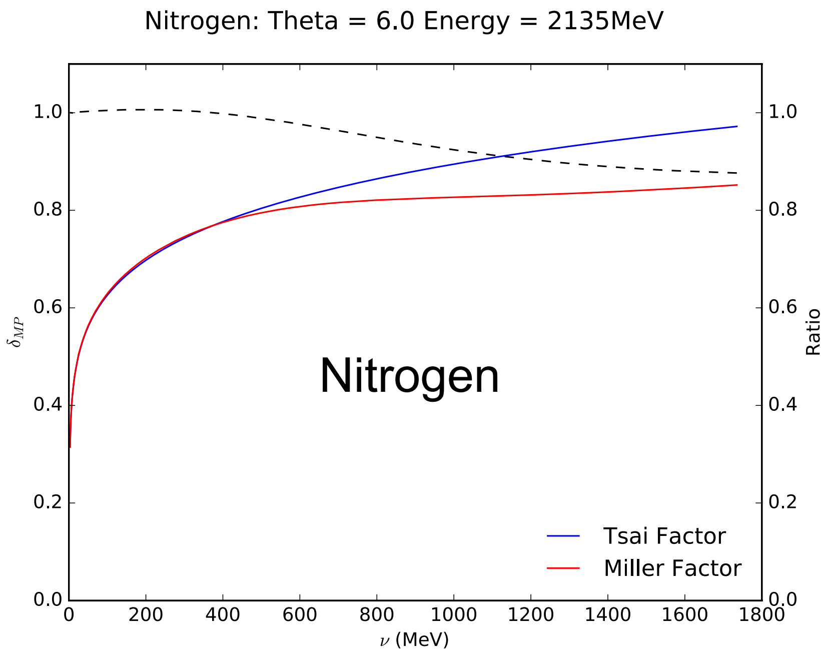
Straggling function

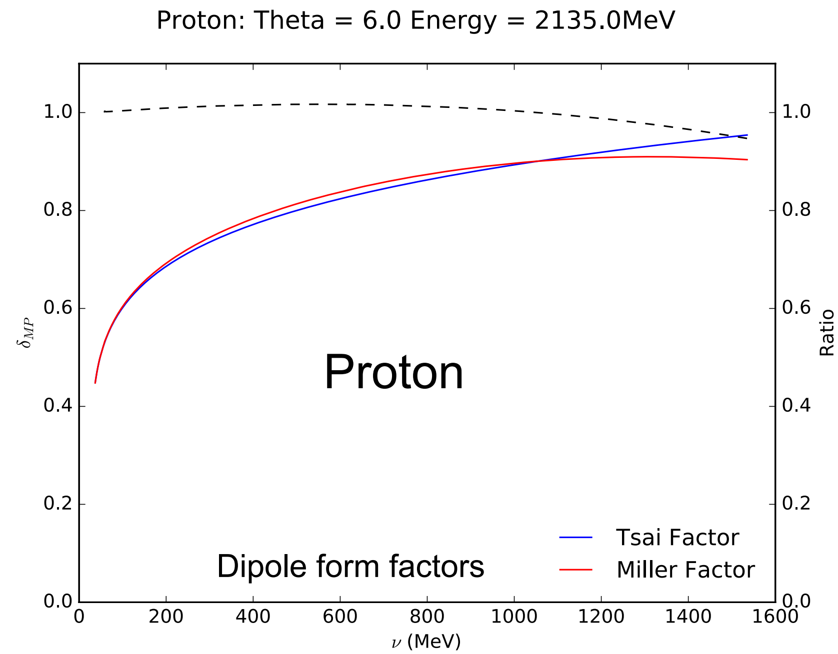
Elastic XS

- *k* is soft-photon limiting energy
- Can compare exact equation to approximate equation to determine *k*/MP contribution

MULTIPLE SOFT-PHOTON CORRECTION COMPARISON

saGDH kinematics:





- Big difference at large *v* for nitrogen
- Agreement at lower *v* (all photons are soft)
  - Tsai (SLAC PUB) stated that MT factor is correct here so this makes sense
- How does it affect the tail subtraction?

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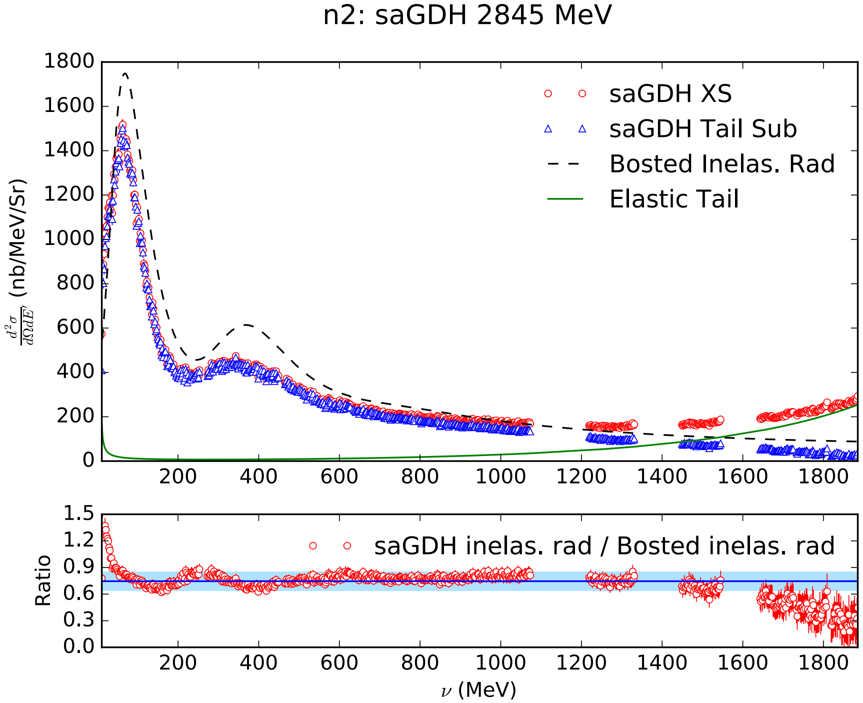
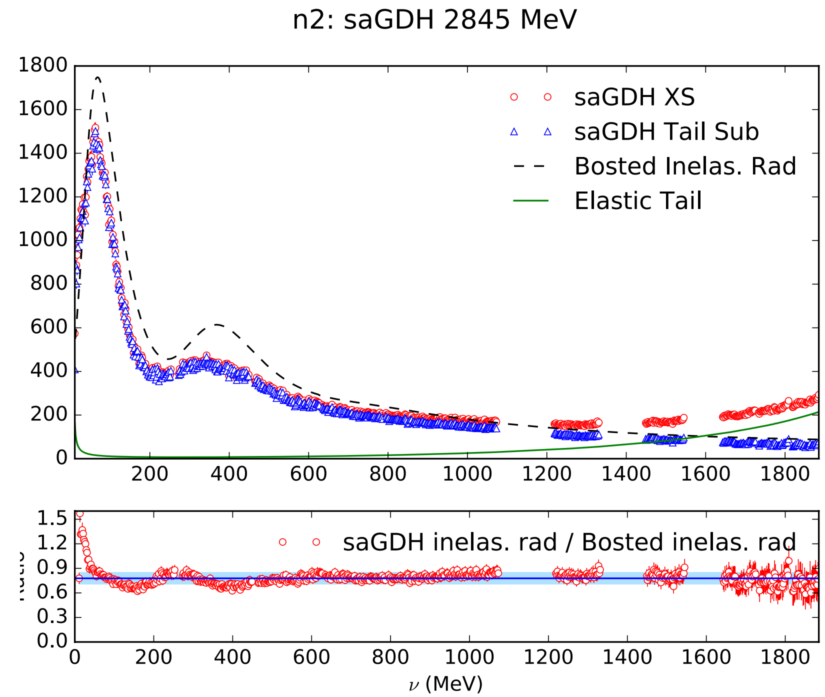
SAGDH N2: 2845/6°

Tsai MP Correction



Miller MP Correction



- Assume MP correction also holds/applies to internal elastic tail
- Same effect anywhere the tail is large
- Use Miller factor for final analysis
- Model: P.E. Bosted, V. Mamyan, "Empirical Fit to Electron-Nucleus Scattering", arxiv.org/abs/1203.2262 (2012)

SUMMARY OF ELASTIC TAIL RESULTS

Total Elastic Tail Systematic Error for saGDH:

- **2%** for loop diagrams/first Born approximation/internal tail integration/soft-photons/ energy-peaking approximation
  - Estimate soft-photon systematic error by looking at factor's sensitivity to form factor fit and value of equivalent radiator for internal contribution
  - Energy-peaking approximation systematic is estimated in Tsai's SLAC PUB
- **4-15%** for form factor error (comparing world data to fit)
  - Break it up into internal and external contributions
    - External limited to world data within Q^2 range of data
    - Internal $1/Q^4$ weighted over all of world data
- **1% and less** for choice of external straggling function
  - Compare MT and Miller external straggling functions
  - Apply bin-by-bin
- Elastic tails calculated using monte-carlo
  - Takes into account acceptance/extended target effects
  - V. Sulkosky, "Update on Corrections to Radiative Tails for E97-110", E97-110 Tech-Note (2015). http://hallaweb.jlab.org/experiment/E97-110/tech/punchthru_update.pdf

INELASTIC RADIATIVE CORRECTIONS

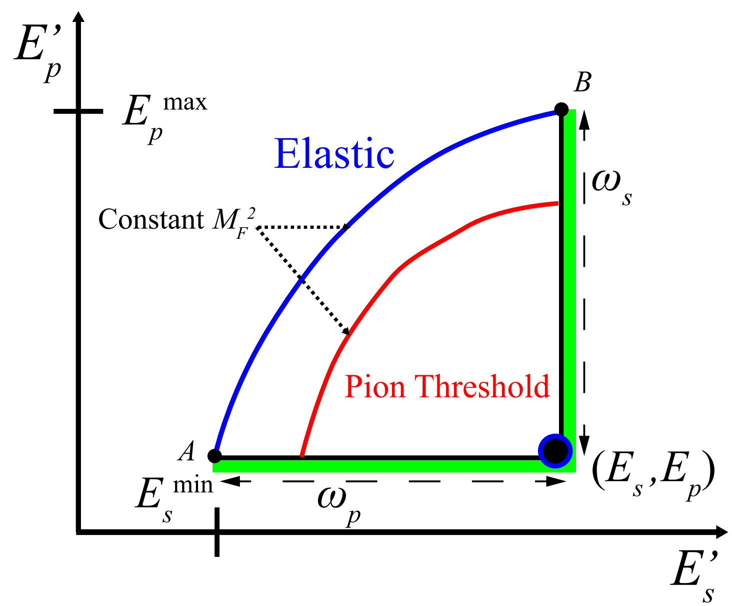
Inelastic continuum can be regarded as a summation of many discrete levels…

Some errors carry over from the elastic tail so don’t redo them:

- ‘Fbar’ (higher order virtual photon corrections)
- Energy-peaking approximation used in evaluating external corrections
- Born approximation

New potential sources of error

- Angle-peaking approximation in internal bremsstrahlung
- Soft photon correction factor
- Interpolation/extrapolation error on unfolding procedure
  - Including using a model as source for lowest energy setting to RADCOR input



INELASTIC INTERNAL BREMSSTRAHLUNG

Full expression of the inelastic internal bremsstrahlung

- Inelastic isn't limited to $W = M_T$ so the inelastic internal contribution is the elastic internal tail integrated over all possible M_f:

$$\frac{d\sigma_r}{d\Omega dE}(\omega > \Delta) = \frac{\alpha^3}{2\pi}\frac{E_p}{E_s M_T}\int_{-1}^{1} d(\cos\theta_k)\int_{\Delta}^{\omega_{\max(\cos\theta_k)}} \frac{\omega d\omega}{q^4} B_{\mu\nu}T^{\mu\nu}$$

Contains the inelastic structure functions

Soft photons

$$\frac{d\sigma_r}{d\Omega dE}(E_s, E_p) = e^{-\delta_r(\Delta)}\tilde{F}(q^2)\frac{d\sigma}{d\Omega dE} + \frac{d\sigma_r}{d\Omega dE}(\omega > \Delta)$$

Unradiated XS

- *Δ* parameter avoids a divergence at $\omega = 0$
  - Slightly tweaked version of (B.6) of MT, but it keeps choice of fbar consistent across calculations (See B. Badalek *et al*. arxiv:hep-ph/940328v1 (1994).)
- Structure functions evaluated at most probable energy loss kinematics
  - Takes into account photons radiating away energy

INELASTIC PEAKING APPROXIMATION

The equivalent correction in the angle-peaking approximation is

- Dropping the soft-photon terms from the integrals. SLAC-PUB-848 (1971) (C.23)

$$\frac{d\sigma_r}{d\Omega dE}(E_s, E_p) = \tilde{F}(q^2)\Bigg[\left(\frac{R\Delta}{E_s}\right)^{bt_r}\left(\frac{\Delta}{E_p}\right)^{bt_r}\frac{d\sigma}{d\Omega dE}(E_s, E_p)$$

Only internal radiation

$$+\int_{E_p+\Delta}^{E_{p\max}} dE'_p \frac{d\sigma}{d\Omega dE}(E_s, E'_p)\frac{bt_r}{2(E'_p - E_p)}\phi\left(\frac{E'_p - E_p}{E'_p}\right)$$

$$+\int_{E_{s\min}}^{E_s - R\Delta} dE'_s \frac{d\sigma}{d\Omega dE}(E'_s, E_p)\frac{bt_r}{2(E_s - E'_s)}\phi\left(\frac{E_s - E'_s}{E_s}\right)\Bigg]$$

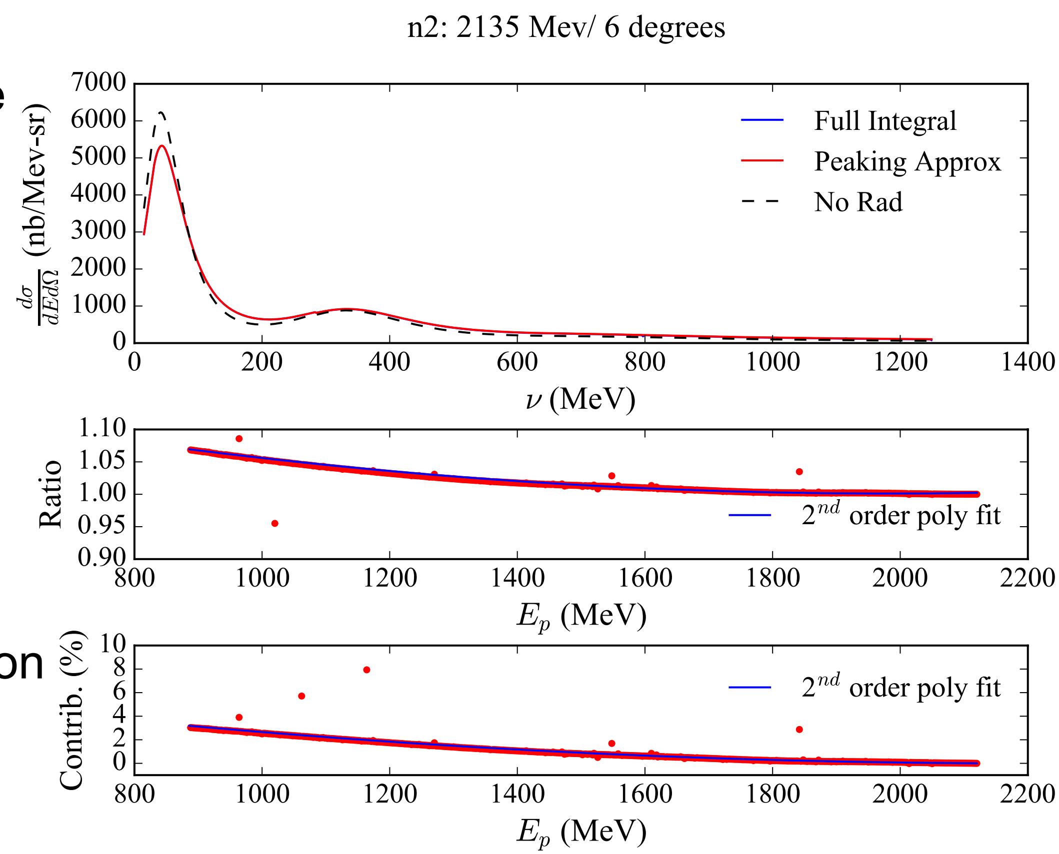
Angle peaking approximation is used because:

- Significantly faster (<1 min to run compared to multiple hours for full integral)\
- Majority of photon's are emitted in same direction as incoming/outgoing electrons

INELASTIC INTERNAL BREMSSTRAHLUNG COMPARISON

Use P.E. Bosted and V. Mamyan model for comparison

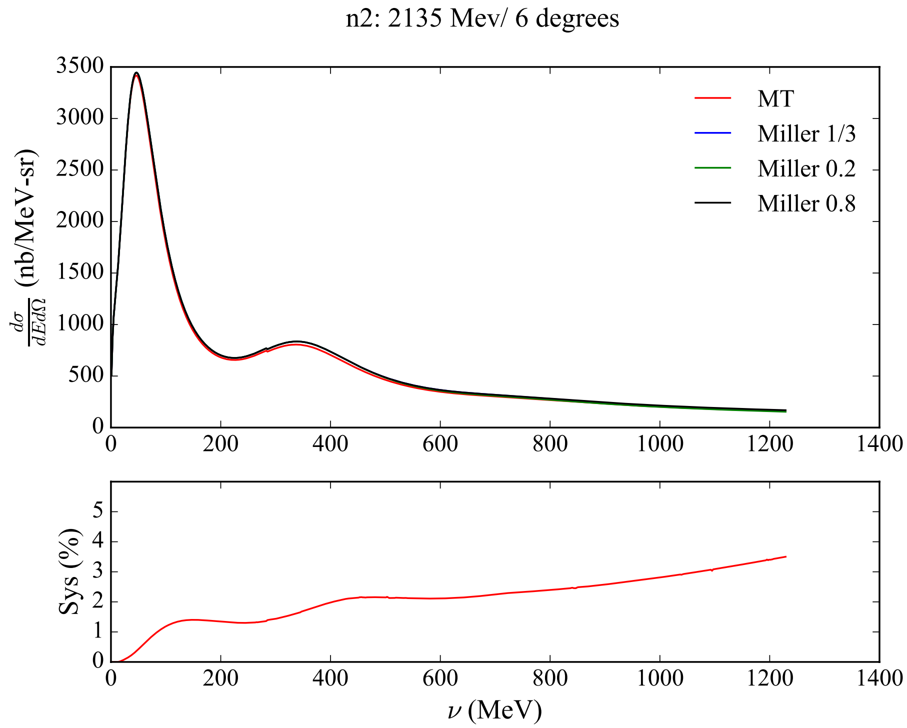
- Full integral computation time ~one week on my desktop
  - Python calculation
  - Proton calculation quicker
- Systematic is contribution to total radiated cross section
  - Comparison between:
    - (Low e + Ext + Int + Coll)
    - (Low e + Ext + Full Int + Coll)
- Assume difference is a function of E_p and fit



INELASTIC MULTIPLE SOFT PHOTONS

Compare soft-photon term from Mo/Tsai to Guthrie Miller

- Separate but similar soft-photon terms for each integral
- Difference in soft-photon terms is smaller for inelastic
    - Sufficient to use an approx value for Miller *k*
- Systematic error is variance between MT term and Miller terms for a range of *k*'s



Use P.E. Bosted and V. Mamyan model for comparison

MT

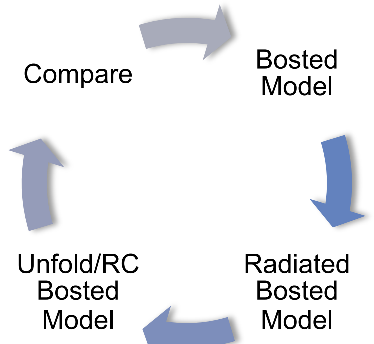
dE$_s$: $$\left(\frac{E_s - E_s'}{E_p R}\right)^{b(t_a+t_r)} \left(\frac{E_s - E_s'}{E_s}\right)^{b(t_b+t_r)}$$

dE$_p$: $$\left(\frac{E_p' - E_p}{E_p'}\right)^{b(t_a+t_r)} \left(\frac{R(E_p' - E_p)}{E_s}\right)^{b(t_b+t_r)}$$

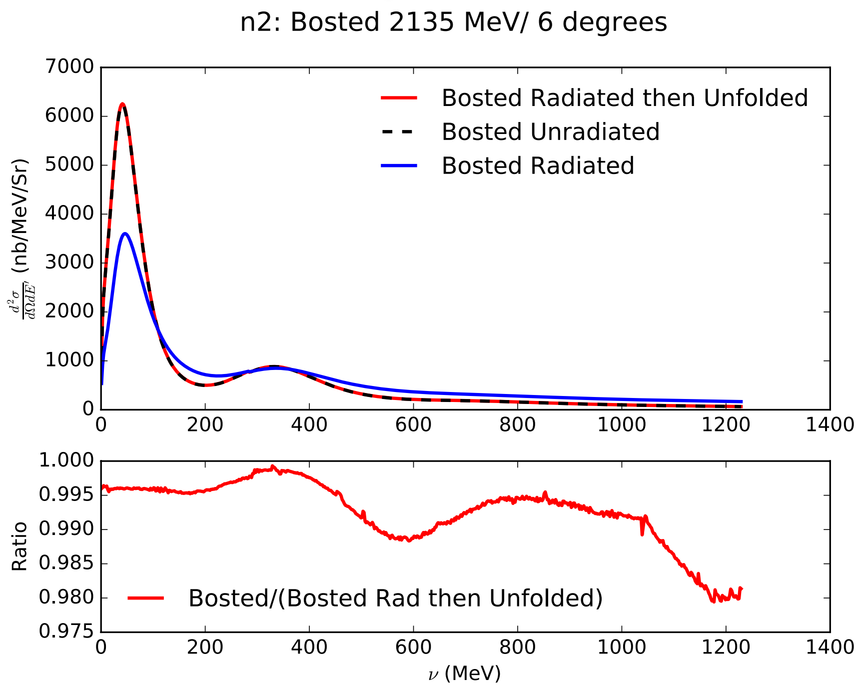
Miller

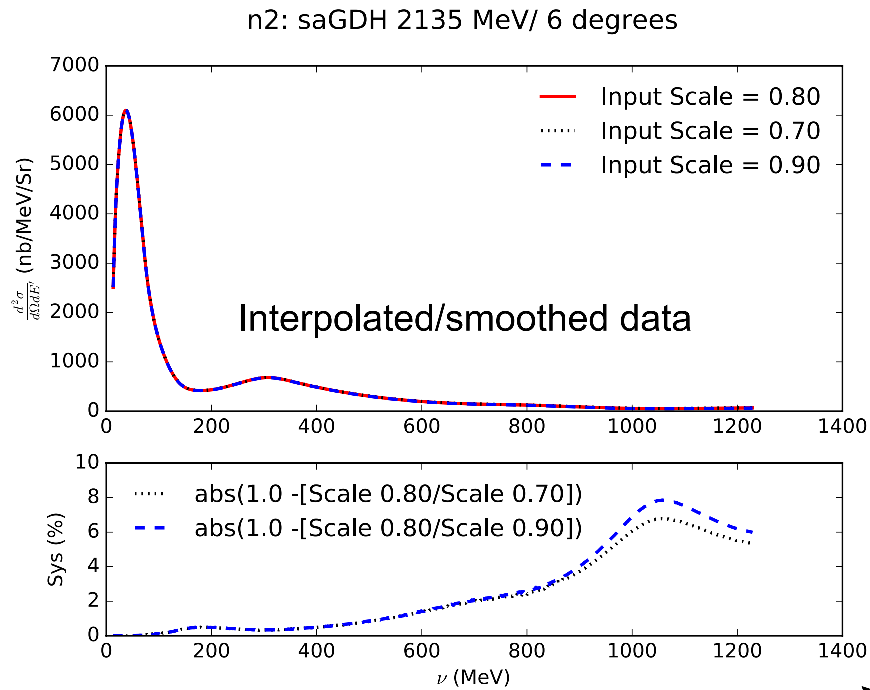
$$\left(\frac{k_1}{E_s}\right)^{b(t_a+t_r)} \left(\frac{k_1'}{E_p}\right)^{b(t_b+t_r)}$$

UNFOLDING SYSTEMATIC ERROR



- Vary lowest energy model input
  - Values for scale guided by comparison of radiated Bosted model vs. data





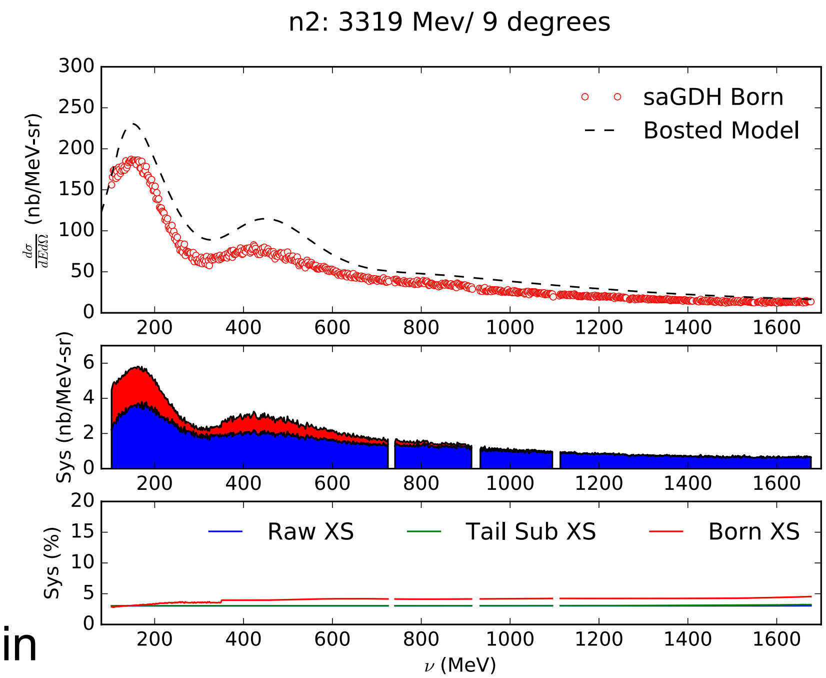
SUMMARY OF INELASTIC RESULTS

Total Inelastic Systematic Error for saGDH:

- **1.5%** for loop diagrams/first Born approximation/energy-peaking approximation
  - Energy-peaking approximation systematic is estimated in Tsai's SLAC PUB
- **<1 – 3%** differences in soft-photon terms
- **<1 – 4%** for angle peaking approximation
- **<1 – 2%** error in the unfolding procedure (driven by extrapolation)
- **<1 – 8%** error from the use of an input model for the lowest extrapolation energy
- Note: variance on unfolding procedure with different scattering angles within the angular acceptance is negligible

CONCLUSION

- Biggest potential systematic error in the Mo and Tsai inclusive radiation scheme comes from handling of soft-photon corrections
  - Error coming from form factor parameterization isn't a limitation of MT
- With requisite hardware possible to replace all peaking approximations with full integrations
  - Did not consider removing energy-peaking approximation in this analysis
- Full results written up and can be found in my tech-note at
  - R. Zielinski,E08-027 (G2P) Tech-Note, https://hallaweb.jlab.org/wiki/index.php/G2p_technotes#E08-027_Technical_Notes (2016).



THANK YOU

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BACK-UP SLIDES

FULL INTERNAL BREMSSTRAHLUNG INTEGRAL

For the elastic tail, the full expression for the internal bremsstrahlung is

$$\sigma_{\text{exact}} = \left(\frac{d^2\sigma}{d\Omega dE_p}\right)_{\text{ex}} = \frac{\alpha^3}{2\pi}\left(\frac{E_p}{E_s}\right)\int_{-1}^{1}\frac{2M_T\omega d(\cos\theta_k)}{q^4(u_0 - |\vec{u}|\cos\theta_k)}$$
$$\times\left(\tilde{W}_2(q^2)\left\{\frac{-am^2}{x^3}\left[2E_s(E_p+\omega)+\frac{q^2}{2}\right] - \frac{a'm^2}{y^3}\left[2E_p(E_s+\omega)+\frac{q^2}{2}\right]\right.\right.$$
$$-2 + 2\nu(x^{-1}-y^{-1})\{m^2(s\cdot p-\omega^2)+(s\cdot p)[2E_sE_p-(s\cdot p)+\omega(E_s-E_p)]\}$$
$$+x^{-1}\left[2(E_sE_p+E_s\omega+E_p^2)+\frac{q^2}{2}-(s\cdot p)-m^2\right]$$
$$\left.-y^{-1}\left[2(E_pE_s+E_p\omega+E_s^2)+\frac{q^2}{2}-(s\cdot p)-m^2\right]\right\}$$
$$+\tilde{W}_1(q^2)\left[\left(\frac{a}{x^3}+\frac{a'}{y^3}\right)m^2(2m^2+q^2)+4+4\nu(x^{-1}-y^{-1})(s\cdot p)(s\cdot p-2m^2)\right.$$
$$\left.\left.+(x^{-1}-y^{-1})(2s\cdot p+2m^2-q^2)\right]\right),$$

EXACT INTERNAL ELASTIC TAIL

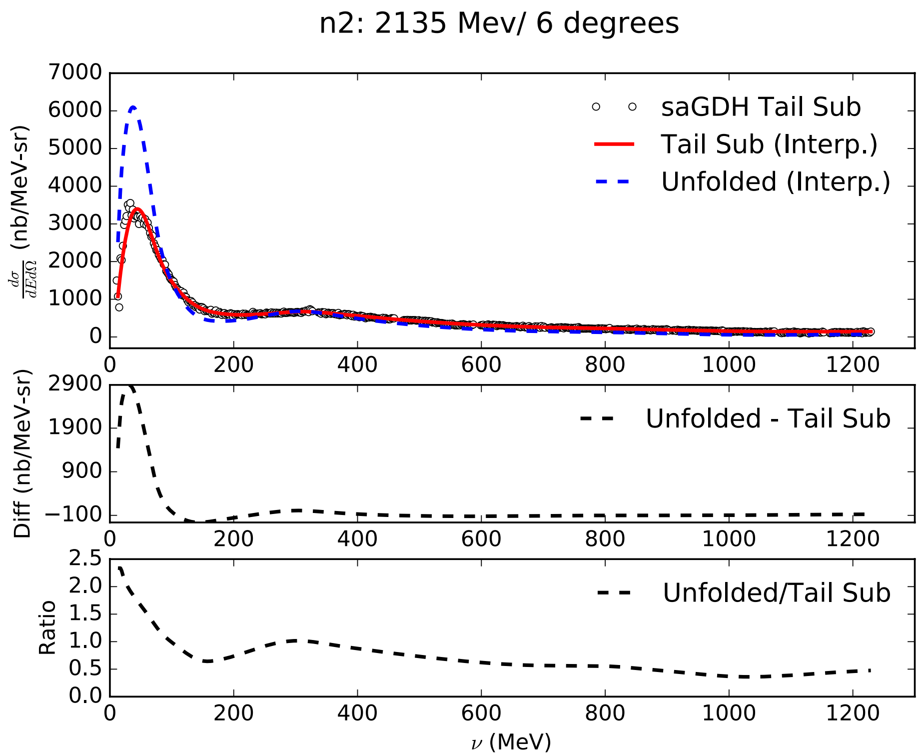
MT give an exact form of the internal bremm. in eq. B5

- Exact means 1st Born approximation and no target radiation
- The equation is an integral with potential for a divide by zero error. MT say to ignore this small point in numerical integration.
  - ROSETAIL has custom integration routine to account for this
  - What about using a more modern integration method?
- Potentially big enough deal that Maximon /Williamson wrote a paper on how to avoid divide by 0. (Paper also helped speed up the calculation)
  - L.E Maximon and S.E Williamson, "Piecewise Analytic Evaluation of the Radiative Tail from Elastic and Inelastic Electron Scattering", Nucl. Instrum. Meth. A258 95 (1987)
- Comparison between ROSETAIL/python integration of B5 (nb/MeV sr):
  - RT: v = 10 MeV -> XS = 2282 && v = 1200 MeV -> XS = 329
  - PY: v = 10 MeV -> XS = 2290 && v = 1200 MeV -> XS = 330
- Systematic for numerical integration is ~0.4%.
  - Systematic contribution to total tail (internal + external) is ~0.2%

A FEW ODDS AND ENDS

- Use difference method to correct quasi-elastic peak and ratio method for rest of spectrum
- Helps control systematic error
  - Systematic errors are applied to the RC correction factor and then propagated in the standard fashion
- Use the Bosted model method for bin-centering
  - Small correction
- Kept the absolute value of the statistical uncertainty constant

Inelastic RC Process



BOSTED RATIO METHOD SYSTEMATIC

Determine systematic by applying Bosted ratio method to data I could unfold

Comparison of the two gives the systematic

- Take weighted average (RC_Bosted/ RC_Unfold) for each spectrum
  - Weight is 1 over the propagated systematic error on the above ratio
- Then average each spectrum to get systematic
- Limit comparison to lowest *W* of spectrum I ultimately want to apply this method to
  - 4209 / 6 degrees:
    - W > 1575 -> Sys = 4.7%
  - 3775 / 9 degrees:
    - W > 1281 -> Sys = 6.5%
  - 4404 / 9 degrees:
    - W > 1641 -> Sys = 4.5%