935 STATISTICAL MECHANICS - Mid-Term. Due: TH, 3/9/2006.

1. This problem is about Information Theory. Assume that all the letters of the English alphabet appear in equal number in Landau and Lifshitz's *Statistical Physics*. How much information in bits does one derive by reading a single letter of the book? Do a calculation to show that one derives twice as much information by reading two very widely separated letters in the book. Is this information more or less than the information derived from reading two adjacent letters? Roughly speaking, how would you expect the length of the book to vary between English, French and Russian editions?

2. This problem is about the Independence of Separated Systems. We assumed in class that for two independent systems in a heat bath,

$$\mathcal{P}(E_1 + E_2) = \mathcal{P}(E_1)\mathcal{P}(E_2) \; . \tag{1}$$

Show that this equation holds for a set of two ideal gases in the thermodynamic limit. Keep track of the fluctuations and show what corrections they lead to.

3. This problem is about the Energy Fluctuations in the Grand Canonical Ensemble. Evaluate $\langle \Delta E^2 \rangle$ in the grand canonical ensemble. Put your answers in the form

$$\left\langle \Delta E^2 \right\rangle = kT^2 C_V + A K_T \left(\frac{\partial E}{\partial N}|_{V,T} \right)^2 \tag{2}$$

where A is to be determined. The answer is different from the answer we obtained in class for the canonical ensemble. Why? If we consider a small volume element inside a large system that is maintained at fixed temperature, fixed number of particles, and fixed volume, what is the appropriate formula for energy fluctuations within the volume element?