935 STATISTICAL MECHANICS - Homework #4. Due: TH, 4/6/2006.

1. This problem is about the Density Matrix. A spinless particle of mass m is confined to a one-dimensional box of length L. Construct the density matrix in the (i) energy, (ii) coordinate and (iii) momentum bases. (Work with the canonical ensemble.)

2. This problem is about Gambling. Gambling may be thought of as a one-dimensional random walk. The probability q of moving to the left and the probability $p = 1 - q$ of moving to the right in the random walk represent the probability that the gambler loses or wins on a specific bet.

   The step size s_n to the left (losing) is equal to the amount bet b_n. The step size s_n to the right (winning) is D times larger, Db_n. The gambler receives D times the amount of the bet when she wins.

   The parameters p, q and D are set by the gambling establishment. The gambler has the freedom to choose the total number of bets N and the amount b_n of each bet, for a given total gambling budget B, where

$$B = \sum_{n=1}^{N} b_n \ . \tag{1}$$

   Assume for this problem that the gambler keeps any proceeds of each individual bet b_n separate from her original budget B and does not reinvest them.

   (a) The "first law of gambling" states that there is no way of varying the number and amount of one's bets within a given budget B that will enhance one's "expected" (average) winnings, $\overline{w}$, defined by

$$\overline{w} = \sum_{n=1}^{N} \overline{s_n} \ , \tag{2}$$

   where $s_n (= -b_n, Db_n)$ is the result of a given bet b_n, and $\overline{s_n}$ is the average result if it could be repeated many times. Calculate $\overline{w}$ and show that it is consistent with the first law of gambling stated above.

   (b) The mean square fluctuation in the expected winnings is given by:

$$\overline{\Delta w^2} = \sum_{n=1}^{N} \overline{\Delta s_n^2} = \left(pD^2 + q - (pD - q)^2\right) \sum_{n=1}^{N} b_n^2 \ . \tag{3}$$

   How do these fluctuations scale with the number of bets N in the special case where all bets are for the same amount $b_n = b$? How do they scale with N for equal sized bets within a given budget B?

   (c) Describe (without extensive calculations) how the gambler can maximize/minimize these fluctuations by using her freedom to choose both N and the distribution of bets $\{b_n\}$ for a given fixed budget B.

   (d) Gambling establishments tend to set the odds against you – they select p, q and D so that $\overline{w} < B$. Describe how the gambler can use the fluctuations to maximize her chances of ending up with more money than she started with.