935 STATISTICAL MECHANICS - Homework #1. Due: TH, 2/2/2006.

1. This problem is about Legendre transforms. Consider the function

$$\Phi(x,y) = A + Bx + Cx^2 + Dx^2y + Exy^2 + Fy^2 + Gy^3 \ . \tag{1}$$

Find the Legendre transform $\Psi(X,y)$ of Φ, where

$$X = \frac{\partial \Phi}{\partial x} \ , \tag{2}$$

explicitly in terms of X and y. Verify explicitly that

$$d\Psi = -xdX + Ydy \ . \tag{3}$$

2. This problem is about thermodynamic manipulations.

   (a) Suppose that a solid sits in a magnetic field, so that

   $$dF = -SdT + HdM \ , \tag{4}$$

   where M is the magnetization, and H is the magnetic field. Derive the relation

   $$\Xi_T = \Xi_S + T\left(\frac{\partial M}{\partial T}|_H\right)^2 / C_H \ , \tag{5}$$

   where

   $$\Xi_x = \frac{\partial M}{\partial H}|_x \tag{6}$$

   is the magnetic susceptibility.

   (b) Find an expression for C_V/V using μ and T as independent variables (all derivatives in terms of μ and T).

3. This problem is about droplet thermodynamics.

   (a) Suppose that we have a small spherical piece of ice in equilibrium in a bucket of water. How much is the melting temperature of the ice reduced by the fact that it is a small spherical drop?

   (b) Letting subscript 1 refer to the drop and 2 refer to the outside water, we have

   $$dF = -S_1dT - S_2dT - p_1dV_1 - p_2dV_2 + \sigma dA \ , \tag{7}$$

   where σ is the surface tension (a constant) and A is the surface area of the droplet. Find how much higher the pressure is within the drop than it is outside of the drop.

   (c) By setting the chemical potential of the material within the drop equal to the chemical potential of the surrounding water, find the small change in coexistence temperature due to the fact that the droplet is of finite rather than inifinite radius. Take p and T to be independent variables.