\[-\frac{g^2}{2\pi^3} \int \frac{d\omega}{\omega^2} \int \frac{d\sigma \, d^2 q}{2\pi i} e^{i\omega r} \frac{\sigma^2}{\sigma^2 + m^2} \]

\[-\frac{g^2}{4\pi^2} \int_0^\infty \frac{d\sigma}{2\pi i} \left( e^{i\sigma r} - e^{-i\sigma r} \right) \frac{1}{\sigma^2 + m^2} \]

\[-\frac{g^2}{4\pi^2} \int_{-\infty}^{\infty} \frac{d\sigma}{\sigma^2 + m^2} e^{i\sigma r} \]

\[V(r) = -\frac{g^2}{4\pi} \frac{1}{r} e^{-mr} \]

"Attractive" Yukawa Potential

range is set by a Compton wavelength: \( \frac{1}{m \omega} = \frac{\hbar}{m c} \)

Relevant to theory of nuclear forces

\[e \rightarrow \text{Nucleons} \]

\[e \rightarrow \text{pions} \]

Yukawa used observationally determined range to estimate pion mass \( \approx 150 \text{ MeV} \)
Quantum Electrodynamics (Q.E.D.)

Recall the Lagrangian:

\[ \mathcal{L}_{\text{QED}} = \bar{\psi} \left(i \not{D} - m\right) \psi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \]

\[ D_\mu = \partial_\mu + ie A_\mu \]

\[ e = -1e \] (charge of electron)

\[ F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \]

Q.E.D. based on:

- \( U(1) \) gauge symmetry: \( \psi \rightarrow e^{i \frac{e}{2} \phi} \psi \)
  \[ A_\mu \rightarrow A_\mu - \frac{1}{e} \partial_\mu \phi \]

- Renormalizability:
  no operators with dimension \( > 4 \).

\[ \mathcal{H}_{\text{int}} = \int d^4x \ e \bar{\psi} \gamma^\mu \gamma^5 A_\mu \]

Very similar to Yukawa theory!!

But has complicating issues:

- photon \( A_\mu \) is massless \( \rightarrow \) infrared issues

- gauge invariance \( \rightarrow \) redundant or complicates derivation of Feynman rules
One way to think of how problems arise:

A_0 does not appear in L.

Hence "momentum" conjugate to A_0 is not there.

Contradicts \[ [A_0(x), A_0(y)] = i\hbar \delta^{(4)}(x-y) \]

Path integral formulation of A.F.T. offers elegant solution.

For now, guess Feynman Rules:

\[ \gamma \mu \ ; \ \mu = -ie \gamma^\mu \]

\[ \mu \gamma^\nu = -i \gamma^\mu \gamma^\nu \frac{g_{\mu\nu}}{g^2 + i\epsilon} \]

Extended photon:

\[ A_\mu (p) = \gamma \mu \gamma^\nu \frac{E_\nu (p)}{p} \]

\[ \gamma p; A_\mu = \gamma \mu \gamma^\nu \frac{E_\nu (p)}{p} \]

Work in Lorentz gauge to maintain manifest Lorentz invariance:

\[ \partial_\mu A^\mu = 0 \]
Recall field equation for $A_\tau$:

\[
\partial_\mu F^{\mu\nu} = 0
\]

\[
\Rightarrow \quad \partial_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu) = 0
\]

\[
\Rightarrow \quad \Box A^\tau - \partial^\nu (\partial_\nu A^\tau) = 0
\]

So in Lorentz gauge:

\[
\Box A_\tau = 0
\]

Hence, each component of $A_\tau$ satisfies \(\Box\) equation!!

Momentum space solutions are of form:

\[
\epsilon_\mu (p) e^{-ip_\tau} \quad p^2 = 0
\]

\[
\epsilon^{\mu \nu} \quad 4\text{-vector}
\]

What about quantized $A_\tau$ field??

\[
A_\tau (x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{r=0}^3 (A_r^\nu \epsilon^{\nu \alpha}(p) e^{-ip_\alpha} + A^{\nu *}_r \epsilon^{\nu \alpha}(p) e^{ip_\alpha})
\]

(Basis of polarization vectors)

\[
\epsilon^\mu = (0, \epsilon_i) \quad p^\mu = (1p_\tau, p_i)
\]

\(\epsilon\) restricted to transverse polaronicks
Large condition: \[ E^\mu p_\mu = 0 \]

\[ E; p = 0 \]

transverse
polarization

If \( p_i \) points along \( z \),

\[ E^x = (0, 1, \pm i, 0) \quad \frac{1}{\sqrt{2}} \quad \text{right-handed} \]
\[ \text{left-handed} \]

Why - \text{Sun in photon propagator} ??

(Problem: because \( g_{\mu\nu} \) is not positive definite)

\( A_0 \) acts on \( s \) of negative norm

Consider how problem arises:

\[ S_q \quad Y \rightarrow Y - \frac{1}{2} (\partial_x A^0)^2 \quad (\text{add } \dot{A}^0 \text{ term}) \]

\[ \left[ A^x(x; \xi), \dot{A}^x(y; \eta) \right] = -i \delta^{x \eta} \delta^{(3)}(x; -y) \]

\[ \left[ A^0(x; \xi), \dot{A}^0(y; \eta) \right] \Rightarrow \text{negative norm states} \]

\[ \text{written by } A^0 \rightarrow A^0 + \]

\[ \text{Met 15,} \quad \left[ A^0_m, A^{0; k} \right] = -2 k_0 (2\pi)^3 \delta^{(3)}(k; -p) \]

\[ S_q \quad \text{1 \(\Gamma\)} = \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2k_0} f(k) A^{0; k + 10} \]
\[ \langle x | x \rangle = \frac{1}{\sqrt{2 \pi \hbar}} \int \frac{d^3k}{(2\pi \hbar)^3} \frac{1 + |k|^2}{2\hbar^2|k|^2} \]

Ouch! Neglect norm

But we can show that these states do not contribute to physical processes!!

Must await path integral formulation to

give careful derivation of photon propagator.